Archimedes' Principle and pressure

Archimedes' principle states that the buoyant force is equal to the weight of the fluid displaced. In equation form, we write this as:

$$F_b = W_{displaced}.$$

If the fluid that is being displaced, then for an object which displaces a volume V, the weight of the fluid displaced is given by:

$$W = \rho_{fluid} Vg,$$

where ρ_{fluid} is the density of the fluid that the object is placed in. In the case of water, $\rho_{water} = 1x10^3 \frac{kg}{m^3}$. V is the volume **of the displaced fluid** and g is the acceleration due to gravity. If the object is completely submerged in the fluid, then

$$V = V_{object}.$$

Let's examine the two obvious cases for an object (such as a metal cylinder) immersed in water and then for a block of wood floating on water. In the case of the cylinder, we find that the buoyant force is given by

$$F_b = \rho_{water} g V_{object}.$$

Now, what will happen for the metal cylinder is that we would see the apparent weight decrease by the buoyant force or

$$\Delta Weight = gV_{cylinder}(\rho_{cylinder} - \rho_{water}).$$

In the case of the wooden block, since the block floats on water, we know immediately that the density of the wooden block is less than that of water. We can answer a different question then for the wooden block, namely how much of the wooden block would be immersed. This fraction, (which is a percentage when multiplied by 100) is obtained by equating the buoyant force to the weight of the block, or

$$F_b = W_{object}.$$

Again, this is true for a floating object.

We can rewrite this in terms of volumes and densities as

$$F_b = \rho_{fluid} g V_{\substack{fluid \\ displaced}} = \rho_{fluid} g V_{\substack{object \\ immersed}}$$

$$W_{object} = \rho_{object} g V_{object}$$

We solve this for the fraction of the block immersed, we find that this is given by:

$$\text{floats} \Rightarrow F_b = W_{object}$$

$$\Rightarrow \rho_{fluid} g V_{\substack{fluid \\ displaced}} = \rho_{object} g V_{object}$$

$$\Rightarrow \rho_{fluid} V_{\substack{fluid \\ displaced}} = \rho_{object} V_{object}$$

$$\Rightarrow \frac{V_{\substack{object \\ immersed}}}{V_{object}} = \frac{\rho_{object}}{\rho_{fluid}}$$

The Procedure

Measure the masses of the block, the aluminum cylinder and also the balloon. In the case of the balloon, I will not fill your balloon with helium until you show me the recorded mass of your balloon. You will also need to measure the volumes of the block, the cylinder and also the balloon.

Standard values:

In the case of the block and the cylinder, obtain the mass from the spring scale.

Object	Mass [kg]	Volume [m^3]	Density [kg/m^3] $= \frac{Mass}{Volume}$
Aluminum Cylinder		$V = \pi r^2 h =$	
Wood Block		$V = L \times w \times h =$	
Balloon			

First, let's make sure you have calculated your volume of your cylinder correctly. Compare your density to the standard value for Aluminum which is given by 3.96 g/cc (note: **1g/cc is equal to 1000kg/m^3 and 1cc is 1x10^{-6}m^3**).

$$\text{Standard } \rho_{aluminum}\left[\frac{kg}{m^3}\right] = \rho\left[\frac{g}{cc}\right] \times 1000 \frac{kg/m^3}{1g/cc} = \_\_\_\_\_\_\_\_\_\_\_\_$$

Calculate your percent error in this measurement:

$$\%E = 100 \frac{\rho_{standard} - \rho_{measured}}{\rho_{standard}} =$$

If this value is too high, measure your cylinder volume and mass again.

In the case of the balloon, you will need to obtain the volume in an unusual way. This is done by filling the balloon with helium and then placing it into a box of peanuts. Fill the box level with the top of peanuts. Removal of the balloon will then leave an empty place in the box. The volume of the balloon is given by the length x width x height of this empty place of the box. Modifications of this technique are often used to find body volume, replacing peanuts by water, particularly in biological applications.

Experiment 1

In the first experiment, you will determine the experimental density of aluminum and then compare it to your previously calculated value. To do this, attach a fishing line to the **aluminum cylinder** and also to a spring scale. Place the aluminum cylinder into a graduated cylinder and measure the weight of the cylinder. Pour water into the cylinder until the aluminum cylinder is completely submerged and record the new weight of the aluminum cylinder from the spring scale.

Weight of Cylinder in air [N]: $W = \frac{m[g]}{1000\left[\frac{g}{kg}\right]} g\left[\frac{m}{s^2}\right] =$ ______________ N

Weight of Cylinder in water [N]: $W = \frac{m[g]}{1000\left[\frac{g}{kg}\right]} g\left[\frac{m}{s^2}\right] =$ ______________ N

Since the cylinder is completely immersed in water, the volume of the fluid displaced is equal to the volume of the cylinder.

Volume of fluid displaced [m^3]: $V_{fluid} =$ __________________ m^3

Buoyant force (calculated) [N]: $F_b = \rho_{water} g V_{fluid} =$ __________ N

The measured buoyant force is given by the difference in the weights of the cylinder when in air as compared to its apparent weight in water.

Buoyant Force (measured)[N]: $F_{b_{(measured)}} = W_{\substack{cylinder \\ in \\ air}} - W_{\substack{cylinder \\ in \\ water}} =$ __________ N

Density of water (measured) [kg/m^3]: $\rho_{water} = \frac{F_{b_{(measured)}}}{V_{cylinder} g} =$ _____________ kg/m^3

The accepted value for the density of water is about 1000 kg/m^3. Determine the percentage error from your measurement and the standard result:

$$\%E = 100 \frac{\rho_{standard} - \rho_{measured}}{\rho_{standard}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Experiment 2

Tie a piece of fishing line around your block and suspend it by your spring scale over a cup of water. It is best to take some care about how you suspend your block since you want a face of the block to be parallel to the ground. Place the block into the **large aluminum cup** and slowly pour water into the cup until the spring scale reads zero. At this point, the buoyant force is equal to the weight of the block. Measure the volume of the block which is not immersed in the water and then calculate the volume of the block which is immersed in the water. From your measurements, determine the fraction of the volume of the immersed block to the volume of the block itself. From this, you can determine the density of the wood. Compare this density to that of oak: ρ_{oak}=0.72 g/cm^3. Be particularly careful when you convert this value to the SI system of units! Note that a cc is a cubic centimeter. You will want to use **your measured value of the density** of water for the following work.

Volume of wood not immersed in water [m^3]:____________________

Fraction of wood immersed in water: $\frac{V_{block} - V_{\substack{not \\ immersed}}}{V_{block}} = \left(\frac{V_{\substack{object \\ immersed}}}{V_{object}}\right) =$ ________________

Density of wood (measured) [kg/m^3]: $\rho_{wood} = \rho_{water_{(std\,value)}} \left(\frac{V_{\substack{object \\ immersed}}}{V_{object}}\right) =$ ________

Density of oak (standard) [kg/m^3]: $\rho_{oak} =$ __________________

Determine the percentage error from your measurement and the standard result:

$$\%E = 100\frac{\rho_{standard} - \rho_{measured}}{\rho_{standard}} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

Do you think that your wooden block is made of oak? Why or why not?

Here are some useful conversions: $1cc = 1x10^{-6}\,m^3 = 1ml$ and $1\frac{g}{cm^3} = 1000\frac{kg}{m^3}$.

Experiment 3

In this experiment, you will determine the lift of a helium balloon. From your measurements, you will be able to determine a value for the density of helium.

The weight of an object in air is a little bit less than it would be in a vacuum owing to the buoyant force from the displaced air. An object which is less dense than air, such as a helium balloon, shows the effect of the buoyant force in remarkable ways in that it can be used to lift object. The lift from a helium balloon is given by

$$\text{Lift} = F_b - W_{helium} = g\left[\left(\rho_{air} - \rho_{helium}\right)V_{balloon}\right]$$

It is the lift that answers the question of how much weight a helium balloon can support.

This equation answers the question "how much weight can a helium balloon lift?" The idea is, for this part of the experiment, to determine this lift by using thread and paperclips. The thread is connected to the helium balloon and then you should attach enough paperclips so that the balloon just barely picks up the paperclips. The thread which we will be using in Coats Rayon decorative 40 weight string which has a linear density given by μ=0.021 g/m. You will use about 3-4 m of this thread. Connect enough paperclips to the system so that the paperclips are suspended but not being pulled upward. A measurement to within fractions of paperclips is accomplished by cutting off portions of the paperclips. Note: the easiest method to measure the lift of the balloon is by putting the paperclips on the scale, and then measuring the weight difference when the balloon lifts the paperclips off the scale.

Length of string [m]:________________ Mass of string [kg]:__________________

Mass of paperclips [Kg]: LIFT=Total weight lifted (except helium):_____________

Buoyant Force = Lift + weight of helium [N]:_____________

Now let me show you how to calculate the density of the helium.

$$F_b = Lift + Weight_{helium} = Lift + \rho_{helium} V_{balloon} g$$

It's pretty easy now to solve this for the density of helium:

$$\rho_{helium} = \frac{F_b - Lift}{g V_{balloon}}$$

We could write this in terms of the density of air:

$$\rho_{helium} = \frac{\rho_{air} g V_{balloon} - Lift}{g V_{balloon}} = \rho_{air} - \frac{Lift}{g V_{balloon}}$$

The Lift is, of course the total weight lifted except for the helium. This is the weight of the clips, the string and the balloon. Calculate now the density of helium. You may use the value for the density of air as 0.0013 g/cm^3 at STP (Standard Temperature and Pressure).

$$\rho_{helium} = \underline{\hspace{4cm}} \frac{g}{cm^3} = \underline{\hspace{4cm}} \frac{kg}{m^3}$$

Helium at STP has the density of ρ_{He}=0.00018 g/cm^3. Find the percentage error between your measurement and the standard value.

$$\%E = 100 \frac{\rho_{standard} - \rho_{measured}}{\rho_{standard}} = \underline{\hspace{4cm}}$$

Don't be too distressed by a rather large percentage error. Our helium is not pure and we have not taken into additional factors involving the balloon here. My primary goal here is for you to see this as yet another application of Archimedes' principle.

Be sure, once again, to work in the SI system of units. Also, answer this: **How many helium balloons of the same size as your balloon will it take to lift a 50 pound child? Show your calculation of this in your writeup.**

How to do this calculation?

$$\frac{1 balloon}{lift} = \frac{x balloons}{weight of child in kg}$$

and then solve for x.

Pressure (read only)

Pressure can be understood in terms of weight in the following way:

Suppose an aquarium is filled with water of density ρ. The cross sectional area of the aquarium is A while the depth is h. The total weight of the water in the aquarium is given by:

$$\text{Weight} = \rho\left[\text{Volume}\right]g = \rho\left[Ah\right]g$$

The pressure at a point in the aquarium is defined as the force per unit area. If you then asked what the pressure was at the bottom of the aquarium, it would be given by:

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\rho g\left[Ah\right]}{A} = \rho g h$$

This actually then tells you how the pressure varies with depth.

Gauge pressure vs. Absolute pressure (read only)

The difference between gauge pressure and absolute pressure boils down to the question of where is the zero on the pressure gauge set at. If you refer to absolute pressure, the zero on the pressure gauge is referenced to the void of space. If, however, you are using gauge pressure, the zero in pressure is set at some arbitrary point, typically the surface of the earth. This would ignore contributions to the pressure from the atmosphere above the surface of the earth. In the example above, since I did not include a term involving the pressure due to the atmosphere, the result is really as a gauge pressure. In order to get an absolute pressure, add the atmospheric pressure to the result above.

Pressure is measured in the SI system in units of Pascals (Pa) which is defined as 1 Pa=1N/m^2. A common unit of pressure often used is, however, mm of mercury since older barometers measured pressure based upon how high of a column of mercury a given pressure could support. This continues to be used in medical applications and the conversion between the two is:

$$1\,\text{mm Hg} = 133.3\,\text{Pa}\,.$$

materials:

peanuts + 2 paper boxes, helium + balloons, rubber bands, wood blocks, spring scales, normal balances, thread, graduated cylinders, large calorimetric cups, small cups (for moving water), metal rulers, digital calipers.