Kinematics in 1 and 2 dimensions
The famous squirt-gun lab.

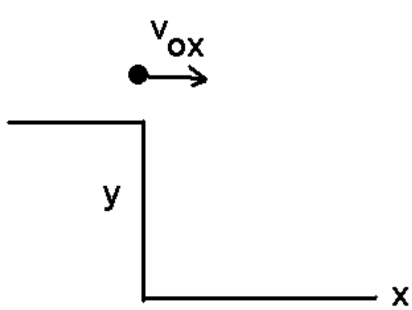
Fluid motion is most correctly discussed using Bernoulli's equation and using this as an example of conservation of energy. Today, however, we are taking a slightly different approach. In particular, I will not be using Bernoulli's equation since it has been my experience that students miss the point that this lab is really about which is 1 and 2 dimensional motion if I do use Bernoulli's equation.

If you treat a stream of water as if it was composed of small non-interacting particles, each with a mass m, then you can imagine that a stream of water is composed of just such particles. Today, the stream of particles will be produced by a squirt gun. Let's now talk about aspects of this experiment which you are more familiar with. The small pellets of water each have an initial velocity $\vec{v}_o$. So long as we direct this either along the x or y direction, we can ignore the vector quality for this discussion. Let's answer the question of what is v_0. The 1-D kinematic equations of motion provide us with the answer here. The third equation is:

$$v^2 = v_0^2 - 2g(\Delta y) \Rightarrow v_0 = \pm\sqrt{2gh}$$

The correct sign here is the positive sign. You will have noted that at the point of maximum altitude, the water has zero velocity.

You can imagine, if you like, that the water gun is shooting out little pellets of water, one after another also in the second experiment.



Suppose a rock is an initial height y_0 above the ground and is thrown with an initial velocity v_{ox} in the x-direction. The question is how far from x_0 will the rock be when it strikes the ground (at y=0)? This is a fundamental type of problem you should be able to work as a result of the physics class but we'll go through the solution here again.

For motion in the x-direction, we have

$$x = x_0 + v_{0,x}t$$

where t is time. For motion in the y-direction, we have:

$$y = y_0 + v_{0,y}t - \frac{1}{2}gt^2$$

Now, suppose that your do the following sequence of experiments: (1) Throw a ball straight up with an initial velocity $V_{0,y}$ (I am using capital letters for v here to reduce confusion. How high does the ball go?

$$V_y^2 = V_{y,0}^2 - 2g(\Delta y) \Rightarrow \Delta y = \frac{V_{y,0}^2}{2g}.$$

Now what this means is that if we measured how high the ball went, we can obtain the initial velocity as:

$$V_{y,0} = \pm\sqrt{2g(\Delta y)}$$

Let's call this $\Delta y \equiv h_{stick}$ since it is measured with a long stick.

Now for the next part of the experiment: you now know how fast water leaves the gun with after this measurement and calculation. It is given by:

$$V_{initial} = \sqrt{2gh_{stick}}$$

Suppose that you now squirt your gun along the x-direction off of the top of the balcony of the library. There is now no initial velocity in the y direction: it is all in the x direction. Further, suppose that the distance from the ground to the gun is given by h_{gun}. How long does it take for the water to hit the ground from this height (which is essentially the same question as how long does it take a ball to fall through this distance when released from rest. The answer to this is given by:

$$y_f = y_0 + v_{0,y}t - \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h_{gun}}{g}}.$$

Now you can answer the question of how far in the x-direction will the stream of water shoot until it hits the ground. This is given by:

$$\Delta x = v_{0,x}t = \left[\sqrt{2gh_{stick}}\right]_{\substack{\text{from first}\\ \text{experiment}}} \times \left[\sqrt{\frac{2h_{gun}}{g}}\right]$$

We can combine these into one single result:

$$\Delta x = \sqrt{2gh_{stick}}\sqrt{\frac{2h_{gun}}{g}} = 2\sqrt{h_{stick}h_{gun}}.$$

This is a simple enough result that you don't need a spreadsheet really to calculate it.

Using your measured h_{stick} and h_{gun}, calculate Δx (this is the theoretical value) before you actually fire your water gun from the library balcony.

$$\Delta x_{expected} = \underline{\hspace{4cm}}\ \text{m}$$

Show me your theoretical calculation before you actually spray water from the balcony. Then you'll fire your gun and obtain $\Delta x_{measured}$.

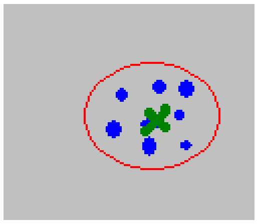
Equipment

Big Stick: In order to measure h_{stick}, I have located a long board. You might consider marking the board with colored chalk to record your measurement here.

Tape measure: In order to measure Δx, use the tape measure I have provided. You'll also use the tape measure ultimately to measure $h_{gun.}$

Water gun: One Black Widow high pressure water gun is needed for obvious reasons.

Colored Chalk: You'll want to mark the water spots that came from your gun. Chalk is probably the thing that you need to do this with. Do it quickly before the water dries. You will want to imagine a circle around the water droplets and take the center of the circle as if it were the point where the water struck the ground.



Procedure

The best location for the height measurements of the water column is in a small corner located at the edge of the library. Fill your water gun with water and then pump it up until the maximum pressure is reached. You will see a bit of water start to drip out of the gun at this point. Align the long stick along the wall and make a quick release **(hint: don't stand up)** on the water gun, allowing the water to strike the wall. Do not move the water gun until the gun's position has been recorded. With the long stick, measure the distance Δh. Reproduce this length with fishing line and cut the line to represent this distance. Give short bursts trying to keep the pressure the same as for experiment 1. You will want to count how many pumps were required for experiment 1 and keep the water level the same (the guns have an indicator on them, and additional water is available).

Next, one lab partner should go to the balcony of the library with the water gun. Measure the height of the gun above the roadway. Align the nozzle of the gun with the edge of the library so that the water shoots out without any initial y velocity. When the gun is fully charged again, release a short burst along the road. Your lab partner should mark with colored chalk the point where the water hits the road. Measure this distance from the building to the chalk mark with the measuring tape. You will note that the water lands in a dispersion pattern. You probably want to measure the center and about 2/3 of the way from the center to the droplets of water represent the +- experimental error.

You should do the entire experiment three times since a small amount of wind can disturb your results. Hint: wait until there is no wind to shoot the gun.

After you complete your experiment, be sure to empty the water guns and observe the trajectory which the water takes once it leaves the gun. The kinematic equations of motion clearly predict this type of motion as you know from class.

Analysis

	Experiment 1	Experiment 2	Experiment 3
h_{stick} (m) (1 time)			
h_{gun} (m) (1 time)			
Δx (m)			
Δx (m) expected			
% difference			

You may calculate the % error by:

$$\%\text{difference} = \frac{\text{expected-measured}}{\text{expected}} \text{x}100 .$$

You may notice that your error is on the order of 15% or so. You should discuss some reasons that this might be the case in your analysis. **Your analysis should also repeat the derivation that I presented on pages 1 and 2 in order to determine the distance the water travels.**

Conclusion

Your conclusion should discuss the equations of motion which were applicable here. You should also make a sketch of the trajectory which the water takes leaving the gun.

Analysis via Bernoulli's Equation

Bernoulli's Equation states: $P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$, where P is pressure (in Pascals), ρ is density in kg/m^3, v is velocity in m/s, g is the acceleration due to gravity (g=9.8 m/s^2) and h is height in meters. Now, we can put this in the form of something a bit more familiar if we multiply both sides of the equation by a volume. What this would then state is that work + kinetic energy + potential energy is equal to a constant. But how can this help us with today's lab?

In today's lab, you are going to experiment with a water gun (in fact, you'll use the Storm 600 water gun). The idea is as follows: you will fill the water tank and then apply the maximum pressure to the tank by pumping the gun. You will then release the trigger. Just after the water exits the nozzle of the gun, you will have a constant pressure (almost) exerted on the column of water until the water reaches its maximum height. If we then remove pressure effects from Bernoulli's equation, we have the following result:

$$(1/2)\rho v_1^2 + \rho g h_1 = (1/2)\rho v_2^2 + \rho g h_2 ,$$

where the 1 subscript refers to water at the nozzle and the 2 subscript refers to water at its maximum altitude. We can also simplify this equation further. Let h_2-h_1=Δh which is the change in altitude of the water. Furthermore, let's divide by ρ on both sides of the equation. You can then easily show from this that the velocity and the height of the water column are related by:

$$v_1 = \sqrt{2g(\Delta h)} .$$