

Example 7.8 (page 306)

A wheel which is free to rotate and of radius b with non-conducting spokes has λ on it's rim. Initially a magnetic field \vec{B} is directed along the $+z$ axis of the wheel out to a radius a ($a < b$). The magnetic field is then switched off. What happens?

According to Faraday's law, we have:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} : \oint \vec{E} \cdot d\vec{L} = -\frac{\partial \Phi_M}{\partial t}$$

The magnetic flux beforehand is given by $\Phi_M = \iint \vec{B} \cdot d\vec{A} = B(\pi a^2)$

The induced emf is then given by:

$$\text{emf} = -\pi a^2 \frac{dB}{dt}$$

The emf is also given by:

$$\oint \vec{E} \cdot d\vec{L} = E(2\pi b) \Rightarrow E(2\pi b) = -\pi a^2 \frac{dB}{dt}$$

The force on the charges on a segment of the rim will be given by:

$$dF = \lambda E b d\phi$$

Noting that the total vector force on the rim is zero.

The torque applied to the rim is then given by: (using $\vec{\Gamma} = \vec{R} \times \vec{F}$)

$$\vec{\Gamma} = \int_0^{2\pi} \lambda E b^2 d\phi = 2\pi b^2 \lambda E$$

But the electric field is given by: $E = \frac{a^2}{2b} \frac{dB}{dt}$

so the torque is given by:

$$\Gamma = \pi \lambda b a^2 \frac{dB}{dt} = \pi \lambda b a^2 \frac{dB}{dt}$$

The torque results in a time rate of change in the angular momentum:

$$\Delta L = \int_0^t \Gamma dt = \pi b \lambda a^2 \int_B^0 dB = \pi \lambda b a^2 B$$

Since the induced current is in such a direction as to reinforce the magnetic field which is leaving the central region, and $\vec{j} = \sigma \vec{E}$, the force will be in the ϕ direction. The torque will be in the z direction and the will rotate so that the final angular momentum is in the z direction.

Your author wishes to point out here that it was the electric field and not the magnetic field which caused the wheel to rotate. Again, magnetic fields do no work.