Example 7.4: (page 298)

A conducting disk of radius a is rotating with its (constant) angular velocity parallel to an external magnetic field. A circuit is connected to a resistor between the edge of the disk and the central axis. Find the current through the resistor.

Considering positive charges on the disk, the magnetic force on a charge is given by: (let u represent tangential velocity)

$$\vec{F} = q\vec{u} \times \vec{B} \quad .$$

At any point along the disk, the velocity is given by:

$$\vec{u} = \vec{\omega} \times \vec{s}$$

Here, the vectors are given by:

$$\vec{u} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \omega \\ s\cos(\omega t) & s\sin(\omega t) & 0 \end{vmatrix} = \hat{x}(-\omega s \sin(\omega t)) - \hat{y}(-\omega s \cos(\omega t)) + 0\hat{z} = \omega s \hat{\phi}$$

The magnetic force vector is then given by:

$$\vec{F} = q\omega s B \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{vmatrix} = q\omega s B[\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y} + 0\hat{z}] = q\omega B s\hat{s}$$

The conventional current is directed along the s direction. The emf is then given by:

$$emf = \int_{s=0}^{s=a} \frac{\vec{F}}{q} \cdot d\vec{s} = \omega B \frac{a^2}{2} = IR \Rightarrow I = \frac{\omega B a^2}{2R}$$

All in all, I think the approach which formulates this problem based upon the Lorentz force is more transparent than the approach which looks at the changing magnetic flux; an approach which never-the-less will give the same answer.