



$$x=0, x=a V=0; y=0, y=b V=V_0$$

Solution:

Switch to the translated coordinate system which is:

$x'=x$; $y'=y-f$ where $f=b/2$. Solve the problem in the primed coordinate system, losing the ' notations. (i.e. write in terms of b and without ' and f)

$$V(x, y) = (A \sin(kx) + B \cos(kx))(C e^{ky} + D e^{-ky})$$

$$@ x=0, V=0 \Rightarrow B=0$$

$$@ x=a, V=0 \Rightarrow ka = n\pi \Rightarrow k = \frac{n\pi}{a}$$

$$V(-b) = V(b) \Rightarrow C = D$$

$$V(x, y) = \sum_{n=0}^{\infty} A_n \sin\left(n\pi \frac{x}{a}\right) \cosh\left(n\pi \frac{y}{a}\right)$$

$$\int_{x=0}^{x=a} V(x, y) \sin\left(m\pi \frac{x}{a}\right) dx = \sum_{n=0}^{\infty} A_n \cosh\left(n\pi \frac{y}{a}\right) \int_{x=0}^{x=a} \sin\left(n\pi \frac{x}{a}\right) \sin\left(m\pi \frac{x}{a}\right) dx = A_m \cosh\left(m\pi \frac{y}{a}\right) \frac{a}{2}$$

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$$V(x, y) = \sum_{n=1,3,5,\dots}^{\infty} \frac{2aV_0}{n\pi} \frac{\cosh\left(n\pi \frac{(y+b/2)}{a}\right)}{\cosh\left(n\pi \frac{3b/2}{a}\right)} \sin\left(n\pi \frac{x}{a}\right)$$