

### 2.3.5 Electrostatic boundary conditions when crossing a surface charge $\sigma$

Solve the Gauss's law for an infinite plane of surface charge  $\sigma$ , making a sketch and noting the image in the text ( Figure 2.36 on page 88).

$$\vec{E}_{\text{below}} = -\vec{E}_{\text{above}}$$

From the Gauss's law solution, we have the results, in fact:

$$\vec{E}_{\text{below}} = \frac{-\sigma}{2\epsilon_0} \hat{z} : \vec{E}_{\text{above}} = \frac{\sigma}{2\epsilon_0} \hat{z}$$

So the discontinuity is :

$$\vec{E}_{\text{above}} - \vec{E}_{\text{below}} = \left( \frac{\sigma}{2\epsilon_0} - \frac{-\sigma}{2\epsilon_0} \right) \hat{z} = \frac{\sigma}{\epsilon_0} \hat{z}$$

Since the electric field is only in the z direction, in fact we call this the “normal” E or, the boundary condition for the electric field is:

$$\Delta E_{\perp} = \frac{\sigma}{\epsilon_0}$$

Upon crossing a surface the tangential component of E is continuous.

$$\Delta E_{\parallel} = 0$$

Even if on one side, the electric field is zero.

Why? Consider a square loop as shown in figure 2.37 page 89 (which is correct).

Then:

$$\oint_{\text{closed loop}} \vec{E} \cdot d\vec{s} = E_{\parallel \text{above}} l + 0 - E_{\parallel \text{below}} l + 0 = 0 \Rightarrow E_{\parallel \text{above}} = E_{\parallel \text{below}}$$

No bets though for other than static electric fields (and these electric fields must arise directly from charge distributions) here.

The combination of these two boundary conditions gives:

$$\vec{E}_{\text{above}} - \vec{E}_{\text{below}} = \frac{\sigma}{\epsilon_0} \hat{n}$$

We can calculate the potential difference upon crossing a boundary:  
Take a perpendicular path from  $z=-a$  to  $z=+a$  through the plane:

$$V_{\text{above}} - V_{\text{below}} = -\oint \vec{E} \cdot d\vec{s} = \frac{-\sigma}{\epsilon_0} a + \frac{\sigma}{\epsilon_0} a = 0 \Rightarrow V_{\text{above}} = V_{\text{below}}$$

The potential is continuous. Since, however, the electric field comes from the gradient of  $V$ , the discontinuity in the gradient of  $V$  is:

$$\vec{\nabla} V_{\text{above}} - \vec{\nabla} V_{\text{below}} = \frac{-\sigma}{\epsilon_0} \hat{n}$$

This can also be written as:

$$\frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = \frac{-\sigma}{\epsilon_0}$$

where the normal derivative is given by:

$$\frac{\partial V}{\partial n} \equiv \vec{\nabla} V \cdot \hat{n}$$