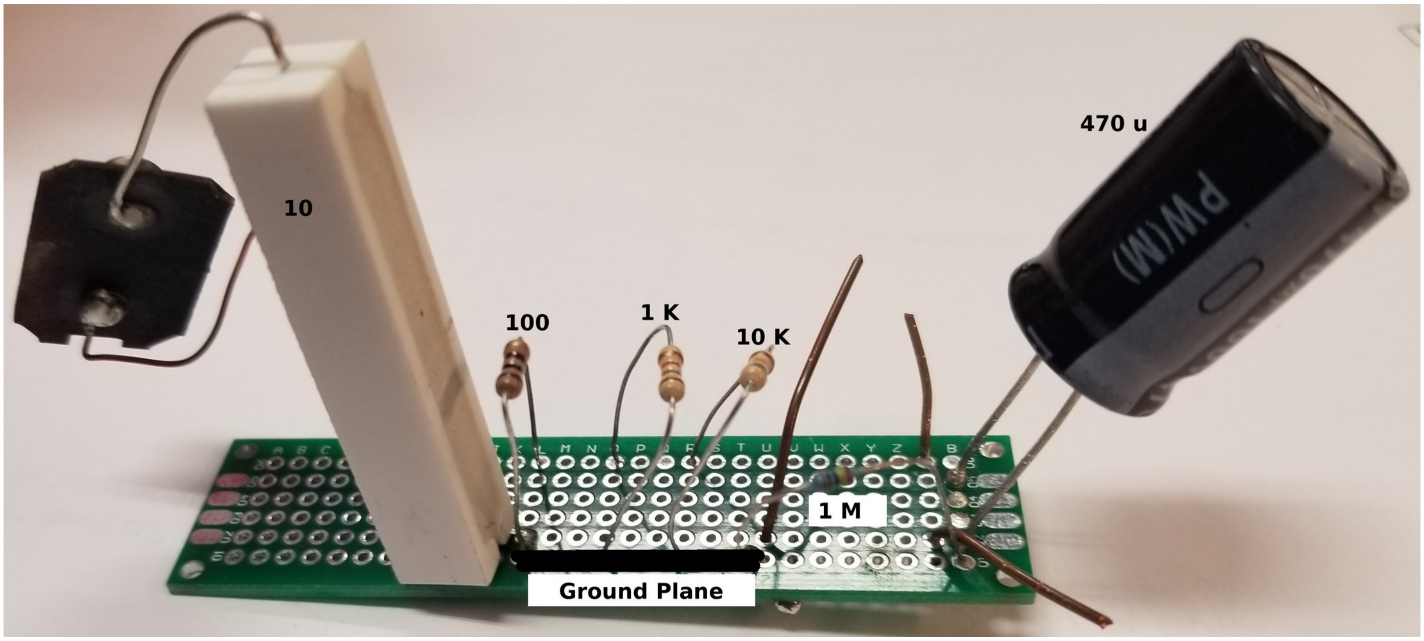
Measurement of the EMF from a source Pandemic Edition R2021
The RC time constant of a series RC circuit.

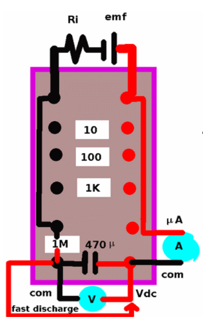
In this lab, you will explore aspects of the EMF from a battery and you will also investigate the time dependence of the series RC circuit.

Image of your circuit for today with elements marked



Part II: The series RC Circuit

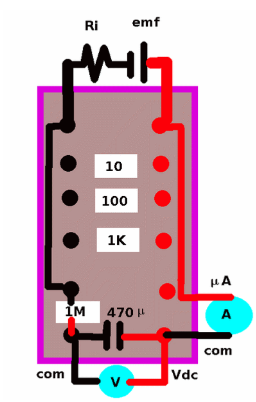
Connections for discharge/charge measurements from the RC circuit



I have the ammeter and the voltmeter already connected for proper charging measurements today. To complete the circuit you will need to connect the μA probe to the positive side of the battery.

To quickly discharge the capacitor, touch the loose wire plugged into the "com" side of the voltmeter to the Vdc side of the voltmeter (or easier, to the copper wire on that side of the capacitor). This shorts out the capacitor. You should read zero volts across the capacitor This is show in the image to the left here.

With the capacitor completely discharged, you are now ready to start charging.



Acquiring the charge data

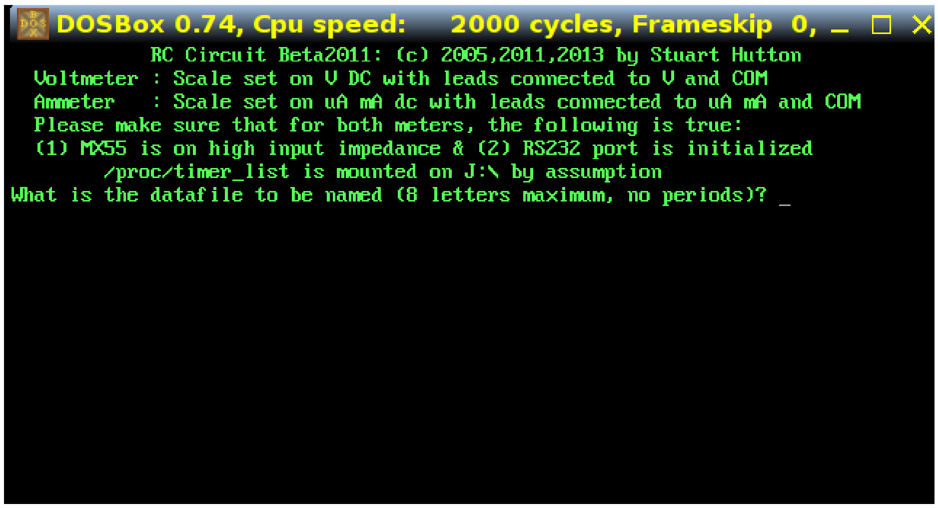
Fast discharge your capacitor with the shorting cable. Make sure the connection between the ammeter and the battery is as I have shown.

Both the voltmeter and the ammeter are set in the high input impedance mode (1000 MΩ). MΩ input impedance, and, of course, the RS232 interface needs to be initialized.

The data acquisition program is named RC-Circuit as shown by the red arrow below. Click only one time on this icon to start the program. If you click the icon twice, the program will fail.

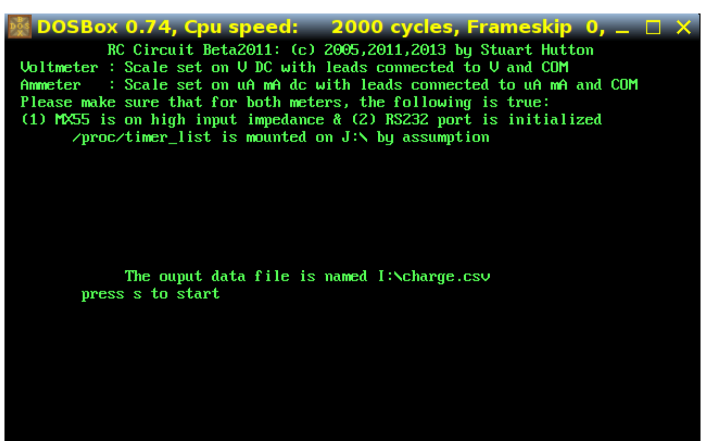
Start your acquisition program. If you get timeout port 1 or timeout port 2, make sure the RS232 interface is connected and that the meter has the RS232 interface properly initialized by pressing pk+/- while cutting meter on. I have already set the meters to be in the high input impedance mode.

After you click on the proper icon, the following screen will appear:



Type your file name following the rules: no spaces, no special characters, less than 8 letters and press enter. I named my file charge. The ".csv" ending is automatically added by the software.

The next screen that appears looks like this:



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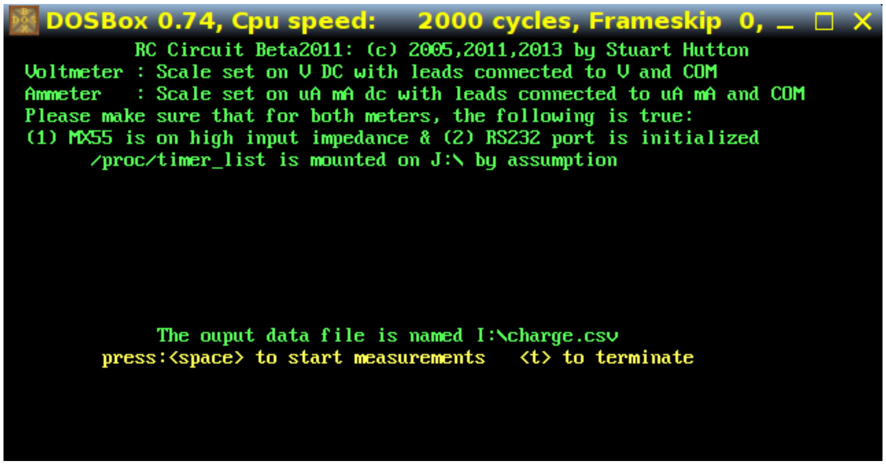
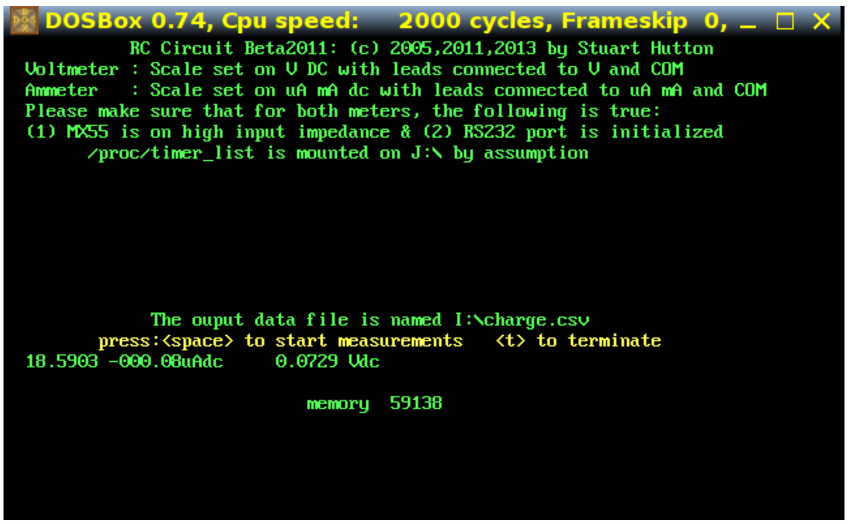
After you press s to start, the following screen will appear:



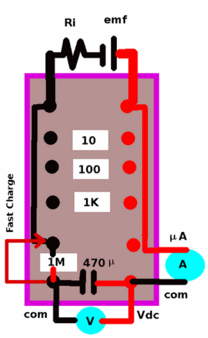
Now you are ready to begin measurements. You should now make sure the capacitor is charging. press the space bar and something similar to the following screen will appear:



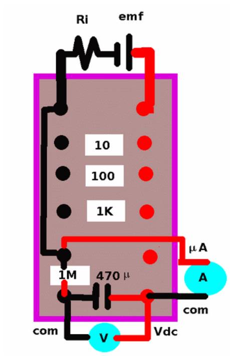
Looking across the screen, you have the time in s, the current in µA and the potential across the capacitor in Vdc. Also included is the system memory which is this old language’s garbage collection technique so that the program does not suddenly freeze.

Now you need to chill for about 5 minutes. Don’t touch the table or any part of the experiment during this time. After 5 minutes (350s) of collection you may press t to terminate. The data file will be written to your I drive.

Acquiring the discharge data



In order to fully charge your capacitor quickly, look at the circuit connections shown below. You want to start with a fully charged capacitor since you are going to be measuring the discharge. Touch the red cable connected to com of the voltmeter to the copper pin on the low side of the 1MΩ resister as I have indicated in the image to the right. The reading on the voltmeter should be about 9 V which is the emf of your battery. If your voltmeter does not read about 9 V, try again. **Write down this voltmeter reading because that is the emf of your battery**. Note that my circuit board already has a ground plane connected to the “low” side of the battery and each resistor so you do not need to worry about the black connection from the resistors back to the battery. My ground plane is simply a solder bead running on the underneath of the circuit board.



Now you need to remove the lead connected to the battery from the ammeter and connect it to the copper wire that you just touched the “fast charge” cable to. This removes the battery from the circuit and results in a closed circuit for the capacitor to discharge through. I show you this connection to the left. After you fast charge the capacitor, you do not need to hurry too much for this connection since the capacitor will still be charging until you actually make this connection. In this discharge case, the current is negative owing to the discharge being in the opposite direction to the charge direction **(you should make sure to note this sign difference from charging circuit on the ammeter.)**

To acquire the discharge data, you start your RC-Circuit program that you have already used, and start taking data. Keep your file name within the rules here. After you start taking data, you should again get about 350 seconds of data. During this time, chill, and do not touch any part of your experiment or table as you did (not) before. After about 350 seconds, press t to terminate.

Analysis of the discharge data

The discharge data should be the cleanest for analysis because you do not have the complications of the battery in this circuit.

The discharge potential across the capacitor should vary as

$$V=V_0e^{-t/\tau}$$

while the discharge current measured should vary as

$$I=I_0e^{-t/\tau}$$

At each point throughout time, we can thus find the resistance as:

$$R=\frac{V}{I}$$

My data plot for voltage vs current is shown below and the absolute value of the slope will be the resistance for an Ohmatic resistance indicated by the very linear relationship here.

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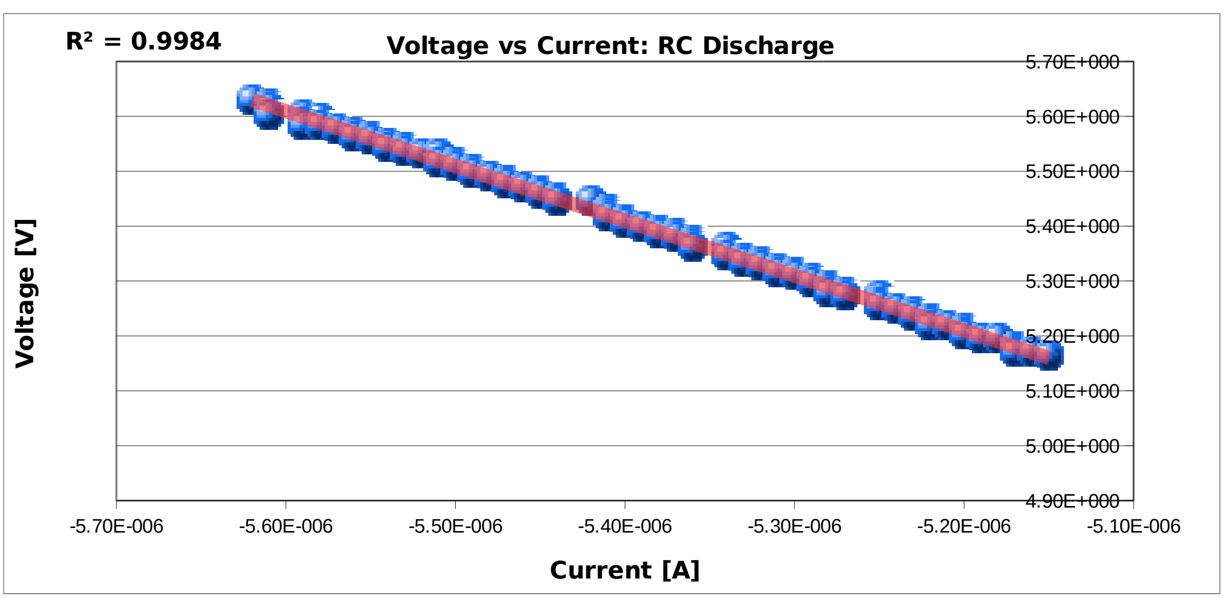


As a note: in fact this slope is not exactly the DC resistance because the potential and the current are both changing in time. For very picky resistances, this could make a difference. I will assume our resistance is not particular sensitive to this.

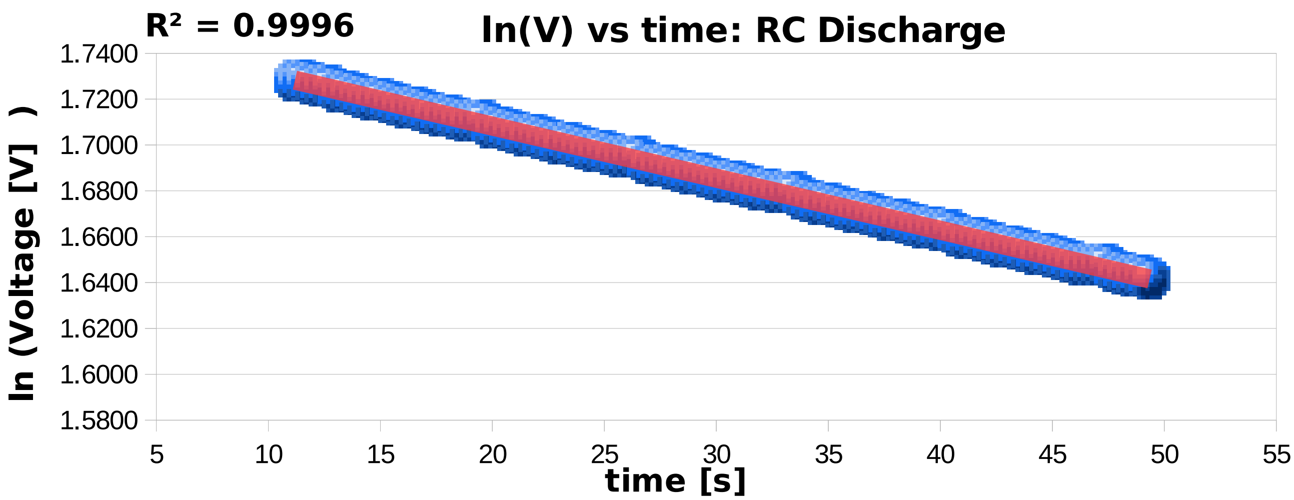
If we take the log of the voltage equation and plot ln(V) vs t, we will have a straight line given by:

$$\ln(V)=\ln(V_0)-\frac{t}{\tau}$$

The intercept is your initial potential difference across the capacitor. If you had started at t=0, this would be the emf of your battery. Since you do not start at t=0, it will not be a good measurement of the emf of your battery. What this will provide is a straight line, with a negative slope which is given by

$$\text{slope}=\frac{-1}{\tau}=\frac{-1}{RC} \quad .$$

Below is a plot of the log of the voltage as a function of time. It is also linear and has a negative slope because the potential across the capacitor decreases as a function of time.



How do we get the separate values of R and C here?

Let's start by getting the value of R by looking at the ratio of V to I:

$$\frac{V}{I}=\frac{V_0 e^{-t/\tau}}{I_0 e^{-t/\tau}}=R \Rightarrow V=IR$$

So a plot of V as a function of I gives the value of R. in fact the resistance is given by the absolute value of the slope. Since it is linear, the resistor is Ohmatic. In fact, we could have done lab 3 by measuring the discharge of a capacitor. My data of the discharge data for voltage vs current looked like what I have shown below. Note that R^2 being very close to 1 means it is a linear relationship.

How to get the value of the capacitance?

We can get the value of the capacitance by looking at the time constant of the circuit. In the helper, I take this from the current although I could have taken it from the voltage also.

The current varies as:

$$I = I_0 e^{-t/\tau} \Rightarrow \ln(I) = \ln(I_0) - t/\tau \quad .$$

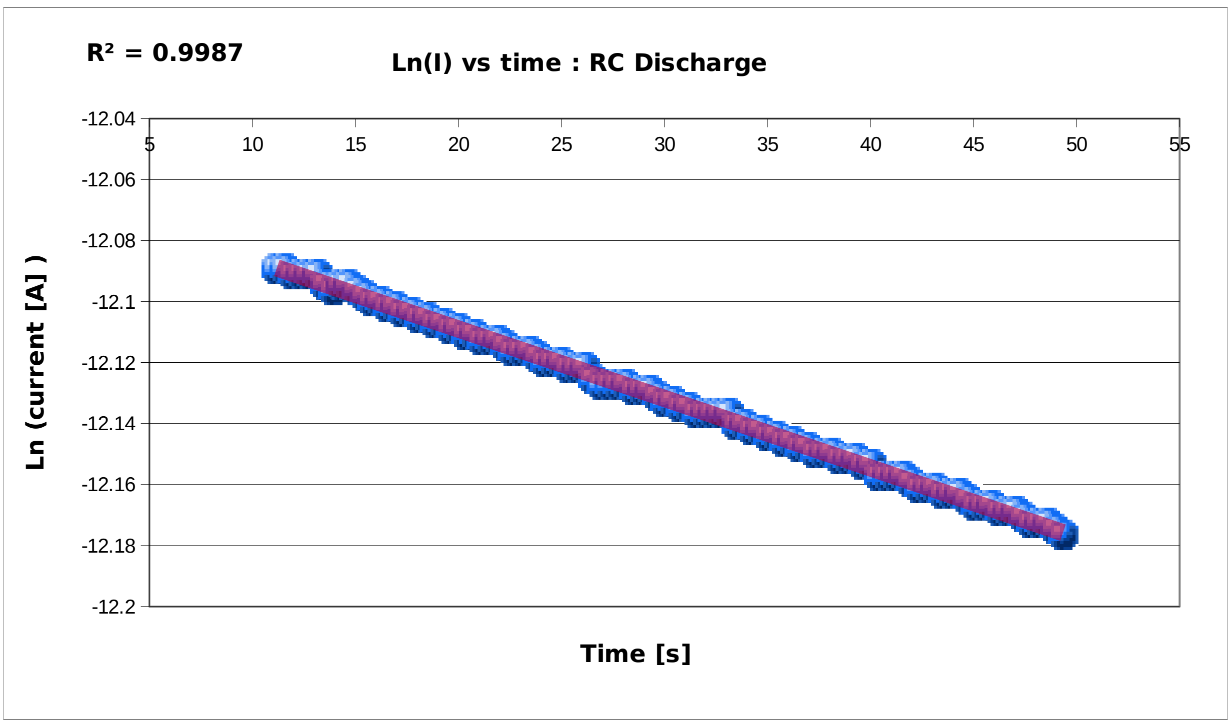
A plot of ln(I) vs t should give us a straight line with a slope of -1/τ. The only little complication here is that the current measured is negative so in fact we need to fit this function:

$$\ln(|I|) = \ln(|I_0|) - t/\tau$$

then everything will work. The helper obtained then τ from this plot. Since the time constant is given by τ=RC, we are able to then find the capacitance from:

$$C = \tau/R.$$

The data for my current looked like what I have shown below. The log is negative because the absolute value of the current is between 0 and 1.



You will also notice that although you did get a measurement of the emf of your battery here, it was really not used in this analysis.

Here is an important question: how do we know, by analysis of the current and the plot of V/I that the voltage follows the same relationship through time as the current? You should answer this.

I suggest that when you cut and paste from the acquired data, select 100 measurements starting after about 15-20 or maybe after even 40 seconds of running, discarding all other data. Paste this into the spreadsheet helper. Then look at and understand the analysis. If you start too close to zero, you will get transient behavior. If you start too close to the end of your data, leakage current from the voltmeters will start to influence things.

Analysis of the charge data

The charging potential across the capacitor in time is given by:

$$V=\mathcal{E}\left(1-e^{-t/\tau}\right)=\mathcal{E}-\mathcal{E}e^{-t/\tau} \quad .$$

But just how can you determine the emf of the battery from the data is the present question. The answer is something like this: as time approaches infinity, the potential difference across the capacitor becomes exactly this emf. This is a little bit more complicated than just taking the logarithm. Since we are not running the experiment to an infinite time here, we in fact will not measure this emf but it would be equal to the potential across the capacitor at infinite time. In reality, the leakage current from the voltmeters will make this method of measurement impractical.

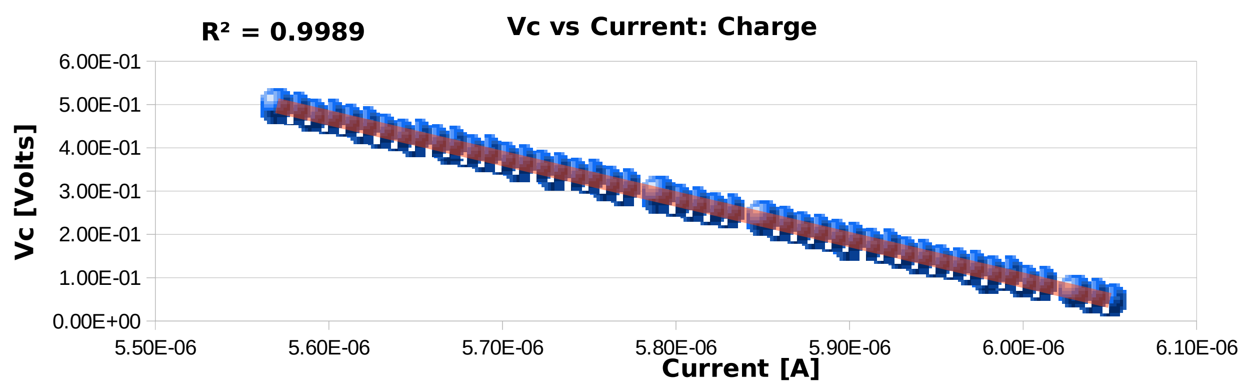
The current through the circuit is given by:

$$I=\frac{\mathcal{E}}{R}e^{-t/\tau}\Rightarrow RI=\mathcal{E}e^{-t/\tau}$$

So we can use this in the voltage equation to obtain:

$$V=\mathcal{E}-RI \quad .$$

This means a linear plot of the voltage as a function of current ought to give a linear relationship with an intercept being equal to the emf of the battery and the slope being the negative resistance. My results are shown below.



If you make a good measurement of the emf (which you did during the discharge measurements) then you can and should compare these two values by the %difference. Do not be surprised if this result is fairly large. This is a hand calculation.

$$\%difference=100x\left[\frac{measure_{charge}-measure_{discharge}}{\frac{1}{2}\left(measure_{charge}+measure_{discharge}\right)}\right]$$

Measurement of the capacitance is done by fitting the logarithm of current to time. For this reason, you need to make sure your data does not contain zero values for the current. If it does, delete those points as these are probably the result of noise in the measurement (i.e. touched the experiment). The time dependence of the current for charging is given by:

$$I=\frac{\mathcal{E}}{R}e^{-t/\tau}$$

If we take the logarithm of both sides, we obtain:

$$\ln(I) = \ln\left(\frac{\mathcal{E}}{R}\right) - \frac{t}{\tau}$$

A plot of ln(I) vs time will then give a straight line with an intercept whose absolute value is the logarithm of $\mathcal{E}/R$ and the absolute value of the slope will $1/\tau$. Since the time constant is RC, we can then find the capacitance from:

$$|\text{slope}| = \frac{1}{RC} \Rightarrow C = \frac{1}{R|\text{slope}|}$$

Here is what my plot looked like for the ln(current) vs time. Note again it is a very linear relationship.

Be sure to report the fit parameters in your report (emf, R, C, and RC). The results for the charging data are not perfect and you may observe significant variations in R and especially C. This would be due to the battery having additional electrical properties that we have not considered in this analysis.

Summary of the lab procedure:

Charge:
Fast discharge the capacitor.
Measure V and I for the charging circuit.
Make a circuit diagram of the charging RC circuit.

Discharge:
Fast charge the capacitor
measure the emf of the battery
Change the ammeter connection for the discharge circuit
Measure V and I for the discharging circuit.
Make a circuit diagram of the discharging RC circuit.

Possible hypothesis (you should phrase in different words).
The voltage, and current for charging and discharging capacitors are described by the exponential functions given above.
The time constant, emf, resistance and capacitance can be obtained from this analysis.