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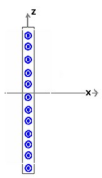
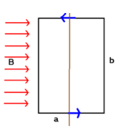
(1) Apply Ampere's law to find the magnetic field from a long wire carrying a current I.

(2) Suppose a wire has a radius a, and current I is uniformly distributed over the area of the wire so that $J=I/(\pi a^2)$. Find the direction and magnitude of the magnetic field both inside and outside the wire.



(3) Suppose that an infinite plane is carrying a current of ***surface*** current density J_s. which is a current per unit length as measured along the z-direction Find the direction and magnitude of the magnetic field on both sides of the plane.



(4) Consider the current loop shown. A current I flows in the circuit. Find the torque on the current loop as a function of angle. The circuit is permitted to rotate about the axis shown.

(5) A rectangular coil of dimensions 0.054 m x 0.085 m consists of 25 turns of wire. The coil carries a current of 1.5×10^{-3} A. Assume that each turn encloses the same area A. (a) Calculate the magnetic moment of the coil and (b)if a magnetic field of 0.55 T is applied parallel to the plane of the loop, calculate the torque on the current loop.(c) Calculate the magnitude of the torque when the magnetic field makes angles of 60^0 and 0^0 with μ.

This worksheet covers 2 important topics: (1) applications of Ampere's law and (2) magnetic moments. **Ampere's law is defined by:**

$$\text{Non-calculus:} \quad \sum_{\substack{\text{path} \\ \text{segments} \\ S_i}} \vec{B}\cdot d\vec{S}_i = \mu_0 I_c$$

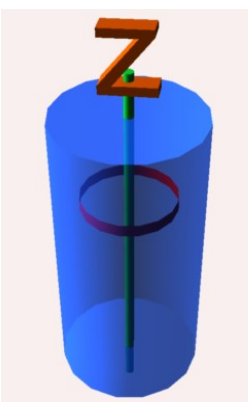
This permits calculation of the magnetic field from currents possessing fairly high degrees of symmetry. The first example that we have seen application of Ampere's law was that of the magnetic field external to a wire carrying a current I. In that case, we found the magnetic field was given by:

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta} \quad .$$

Let's look at the same problem but now in more detail. Suppose the wire had a radius a and the current was evenly distributed over the cross sectional area of the wire. This allows us to define the magnitude of the current density in the wire as:

$$J = \frac{I}{\pi a^2} \quad .$$

Now if we wanted to properly assign the vector quantity to this, we need to look at the cross section of the wire itself. For our present purposes, let the wire be directed in the $+\hat{z}$ direction and also let the current go in this same direction. The vector current density is then given by:



$$\vec{J} = \frac{I}{\pi a^2}\hat{z}$$

It is important to remember the units of this current density:

$$J\left[\frac{A}{m^2}\right] \quad .$$

Now we choose again the same type of Amperian loop with a radius r<a. By symmetry, we then have:

$$\text{Non-calculus:} \quad \sum_{\text{segments}} \vec{B}\cdot\vec{S} = B(2\pi r) = \mu_0 I_c$$

$$\text{Note:} \quad \mu_0 = 4\pi x 10^{-7}\frac{Tm}{A} \quad .$$

The different part of this problem involves the fact that with the Amperian loop inside the wire, we no longer enclose the entire current of the wire with our loop. The current cutting through the wire is given by:

$$\text{Non-calculus:} \quad I_c = \vec{J}\cdot\vec{A} = J(\pi r^2) = I\frac{r^2}{a^2}$$

$$\text{We then have the magnetic field:} \quad \vec{B} = \mu_0\frac{I\frac{r^2}{a^2}}{2\pi r}\hat{\theta} = \frac{\mu_0 I r}{2\pi a^2}\hat{\theta} \quad .$$

Calculus note: if J is not uniform across the diameter of the wire, then you can not necessarily take J outside the integral above. In both cases, the direction comes from the right hand rule for magnetic fields. Here is a slightly more complicated version of this problem: suppose that this is not a solid cylinder but instead a hollow cylinder, with an inner radius d. You will need to define your current density so that on the ultimate inside, there is no current flowing.

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The magnetic moment

Suppose in a planar configuration, current is flowing around a closed area. We can right now, for simplicity, imagine a piece of wire bent into a circular path which is lying in the x-y plane. If the current is circulating counter-clockwise around the loop, the magnetic moment of the current loop is defined by:

$$\vec{\mu}=I\vec{A}$$

where the normal to the area points (in this example) in the +z direction. The right hand rule for determination of the direction of the current loop says to let your fingers curl around the area in the direction in which the current flows. Your thumb points in the direction of the magnetic moment.

I will show you below in a problem that when a magnetic moment is placed in an external magnetic field, a torque will be exerted upon the current loop which is given by:

$$\vec{\tau}=\vec{\mu}\times\vec{B} \quad .$$

Note: it's not exactly the magnetic field that's exerting the torque… it's the action of the current in the magnetic moment producing a magnetic field that does this. As the loop begins to rotate, an entirely new can of worms begins to open and an opposing current will be induced in the loop. These details, however, will need to wait further exploration. For now, we'll not worry about this effect.

In fact, it is possible to talk about the work that happens when this torque acts through an angle. In this case, suppose that the magnetic moment was initially given by: $\vec{\mu}=|\vec{\mu}|\hat{y}$ and the magnetic field was given by $\vec{B}=|\vec{B}|\hat{z}$. The torque on this magnetic moment would be given by: $\vec{\tau}=|\vec{\mu}||\vec{B}|[\hat{y}\times\hat{z}]=|\vec{\mu}||\vec{B}|\hat{x}$.

The change in angular momentum is in the +x direction in this case. This means that the magnetic moment would begin to rotate from the y-axis towards the z-axis (remember the right hand rule for angular momentum from last semester to convince yourself of this fact). I have an animation showing this alignment. The potential energy associated with a magnetic dipole moment in a magnetic field can be expressed as: $U=-\vec{\mu}\cdot\vec{B}$.

For non-calculus people, there is a calculus proof however I think you will just need to accept that this represents the work done.

The work done to orient the loop from an angle $\theta=90^0$ to an angle θ is given by:

$$W=\int_{\theta=90^0}^{\theta}\tau\, d\theta=|\vec{\mu}||\vec{B}|\int_{\theta=90^0}^{\theta}\sin(\theta)\, d\theta=-|\vec{\mu}||\vec{B}|\cos[\theta]_{90}^{\theta}=-\vec{\mu}\cdot\vec{B}=U$$

This expression (although somewhat misleading) is useful in the thermodynamics and quantum mechanics of magnetic dipoles. This says that the potential energy gets lower as the dipole aligns with the magnetic field.

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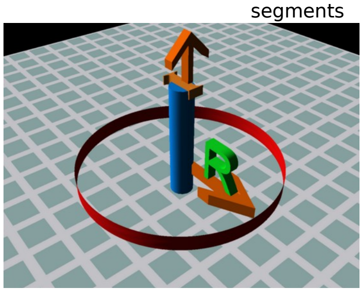
(1) Apply Ampere's law to find the magnetic field from a long wire.
Along a circular path at a radius r from a current element I, B is given by

Non-calculus: $\sum_{\substack{\text{path}\\\text{segments}}} \vec{B}\cdot\vec{S}=\mu_0 I_c$



where I_c is the current passing through the area enclosed by the path. In order to successfully apply this, you must consider a region where B is parallel to or perpendicular to the portion of the curve. The curve ultimately must form a closed path.

Consider an infinitely long wire of negligible radius. Let the wire carry a current I. Use the ***right-hand-rule for magnetic fields*** to find the direction of the magnetic field around the wire. Then, apply Ampere's law to obtain the magnetic field. The answer is $\vec{B}=\frac{\mu_0 I_c}{2\pi r}\hat{\theta}$.

(2) Suppose a wire has a radius a, and current I is uniformly distributed over the area of the wire so that $J=I/(\pi a^2)$. Find the direction and magnitude of the magnetic field both inside and outside the wire.

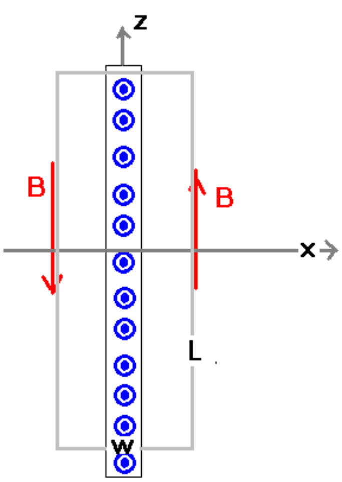
Solution: Inside the wire, $B(2\pi r)= \mu_0 I_c$ and the current inside the wire is $I_c=I\left(\frac{r}{a}\right)^2$.

How do I know this?

$$\text{non-calculus:}\quad I_c=\sum_{\substack{\text{area}\\\text{bits}}}\vec{J}\cdot\Delta\vec{A}=\frac{I}{\pi a^2}\left[\pi r^2\right]=I\left(\frac{r}{a}\right)^2$$

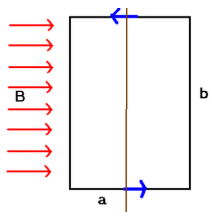
If the path is outside the wire, $I_c=I$. Solve these to get the dependencies for B.

(3) Suppose that an infinite plane is carrying a current of ***surface*** current density J_s. which is a current per unit length as measured along the z-direction Find the direction and magnitude of the magnetic field on both sides of the plane..

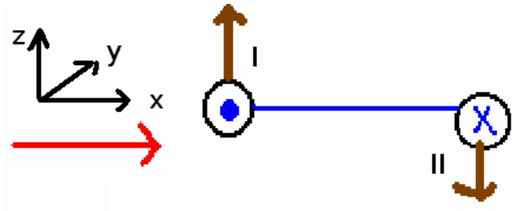


A note about the current density defined in this problem: The current density here has units of $\frac{A}{m}$ and can be thought of as the limiting case of an infinitely wide ribbon cable. I'll use the special s subscript to designate that this is a different type of current density. This problem is of great importance to help form a model for electromagnetic waves.

Solution: on the square path of length L and width W, B is uniform (by symmetry) and in the direction shown (by RHR#2). On the sides, B and L are in the same direction while on the sides marked W, B is perpendicular to W. So then $2B=\mu_0 I_c$. I_c is given by $J_s L$. We can now solve for B which is given by $B=\mu_0 J_s/2$. Note that this is uniform meaning that no matter how far you are from the plane, you get the same value.



(4) Consider the current loop shown. A current I flows in the circuit. Find the torque on the current loop as a function of angle. The circuit is permitted to rotate about the axis shown.



Solution: the net magnetic force on this current loop is zero since the loop is closed. However, in general the torque is not zero. Looking at a sideways view shows the torque most clearly: The brown arrows shows the Lorentz force. As I have this drawn, on the portion marked "I", $\vec{B}=B\hat{x}:I\vec{L}=-Ib\hat{y}$, so that the force on this portion is $\vec{F}_1=-IbB(\hat{y}\times\hat{x})=+IbB\hat{z}$. On the portion marked "II", the force is then given by: $\vec{F}_{11}=IbB(\hat{y}\,x\,\hat{x})=-IbB\hat{z}$. There is thus no net force on the circuit. There is, however a torque on this circuit. We can imagine that the circuit is pivoted about its center. The magnitude of the total torque about this pivot is then given by: $\tau=I\left(\frac{a}{2}\right)bB+I\left(\frac{a}{2}\right)bB=I(ab)B=IAB$ where A is the area enclosed by the loop. We want to express this in terms of the magnetic moment. You can note that the direction of the torque above is in the +y direction and this is also the direction of the change of angular momentum. See

http://www.compchem.org/~shutton/Courses/FA15/250/pdf/Ows17.pdf

for a review of these topics. We can write the magnitude of this in terms of the magnitude of the magnetic moment as $\tau=\mu B$. Note that if the magnetic moment points in the +z direction, then since $\hat{z}\times\hat{x}=\hat{y}$, we should write the torque more generally to incorporate any angle between the two vectors as $\vec{\tau}=\vec{\mu}\times\vec{B}$. This is, then the torque on any planar current loop in a uniform magnetic field.

Note1: The direction of a planar magnetic moment is given by another right hand rule: curl your fingers in the direction of the (conventional) current around the loop. Your thumb points in the direction of the magnetic moment. I have an animation of this.

Note 2: The magnitude of the torque can be written as: $\tau=\mu B\sin(\theta)$ where θ is the angle between $\vec{B}$ and $\vec{\mu}$.

(5) A rectangular coil of dimensions 0.054 m x 0.085 m consists of 25 turns of wire. The coil carries a current of $1.5\text{x}10^{-3}$ A. Assume that each turn encloses the same area A. (a) Calculate the magnetic moment of the coil and (b)if a magnetic field of 0.55 T is applied parallel to the plane of the loop, calculate the torque on the current loop.(c) Calculate the magnitude of the torque when the magnetic field makes angles of 60^0 and 0^0 with μ.

Solution: (a) The magnetic moment of the coil is given by

$$\mu_{coil}=NIA=(25)(0.085\,x\,0.054)(1.5\text{x}10^{-3})=1.72\text{x}10^{-4}\,Am^2(\text{ or J/T}) \ .$$

(b) Applying the magnetic field parallel to the plane of the loop means that it is perpendicular to the magnetic moment of the loop. Thus the torque is given by

$\tau=\mu B\sin(\theta)=9.46\text{x}10^{-5}J$ (but a better unit of torque is, of course, Nm).

(c) at 60^0, $\tau=9.46\text{x}10^{-5}\sin(60)=8.19\text{x}10^{-5}$ Nm. At 0^0, the torque is zero.