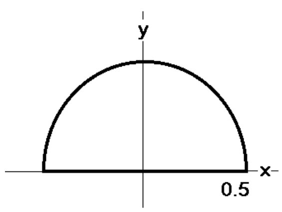
2016	Physics **250**: Worksheet 11	Name _______________

(1) Suppose a proton moves with a speed of $8 x 10^6$ m/s along the positive x axis. If it enters a region where $\vec{B} = -25\hat{y}\,T$, find the direction of the force and the acceleration of the proton. Now, if $\vec{B} = B_0\hat{x}$, what happens? The mass of the proton is $1.672\,621\,71 x 10^{-27}$kg and the charge on the proton is $1.602\,176\,53 x 10^{-19}$C which, interestingly enough, gives a ratio of about $1 x 10^8$.

(2) A wire can be viewed as a path along which charge carriers flow. You can write the following: in the Lorentz Force Law: $q\vec{v} \times I\vec{L}$ where $\vec{L}$ represents the vector length of the wire, and I is the *conventional* current. Now, an important thing to remember is this: if **L** represents a **closed** loop of some arbitrary shape (but, planar), then the net force on the wire loop will be zero if B is uniform.

(a) Suppose a long wire is running along the x-axis and the wire has a length of 1 m. You may assume the wire is symmetrically placed so that it runs from x=-0.5m to x=+0.5m. Furthermore, let the wire carry a current of 1 A towards the positive direction. Let us place this wire in a uniform magnetic field (B) directed along the +Y axis. What is the magnitude and direction of the force on the wire if B=1T?



(b) Now suppose that your wire comes back with a semi-circular loop as shown below. What is the force on the semicircular loop?

(3) Motion of a charged particle in a combined electric and magnetic field. Explain how a *velocity selector* and a Mass spectrometer work.

(4) A singly charged positive ion has a mass of $3.2 x 10^{-26}$ kg. After being accelerated through a potential difference of 833 V, the ion enters a magnetic field of 0.92T along a direction perpendicular to the direction of the electric field. Calculate the radius of the path of the ion in the magnetic field.

(5) A current I=15A is directed along the positive x-axis in a wire perpendicular to a magnetic field. The current experiences a magnetic ***force per unit length*** of 0.63 N/m in the negative y direction. Calculate the magnitude and direction of the magnetic field in the region through which the current passes.

(6) A wire of radius a carries a current I. Find the magnetic field produced at distances R outside the wire.

2016	Physics **250**: Worksheet 11	Name _______________

(1) Suppose a proton moves with a speed of 8x10^6 m/s along the positive x axis. If it enters a region where $\vec{B}=-25\hat{y}\,T$, find the direction of the force and the acceleration of the proton. Now, if $\vec{B}=B_0\hat{x}$, what happens? The mass of the proton is $1.672\,621\,71\,x10^{-27}kg$ and the charge on the proton is $1.602\,176\,53x10^{-19}C$ which, interestingly enough, gives a ratio of about 1x10^8.
Solution:
The Lorentz force on a charged particle is given by: $\vec{F}=q\vec{v}\times\vec{B}$. Here, for the first part of the problem, we have: $\vec{v}=v\hat{x}$. We can thus formulate the cross product:

$$\vec{v}\times\vec{B}=(8.6x10^6)(-25)\hat{x}\times\hat{y}=-2.15x10^8\hat{z}$$

$$\Rightarrow\vec{F}=(+1.6x10^{-19})(-2.15x10^8\hat{z})=-3.44x10^{-11}\hat{z}N$$

The direction of the force is in the -z direction. The acceleration of the proton is given by Newton's laws:

$\vec{F}=m\vec{a}\Rightarrow\vec{a}=\frac{\vec{F}}{m}=\frac{-3.44x10^{-11}\hat{z}}{1.67\,x\,10^{-27}}=-2.12\,x\,10^{16}\hat{z}\frac{m}{s^2}$. Such a large acceleration would not last very long ... energy would rapidly be radiated away.

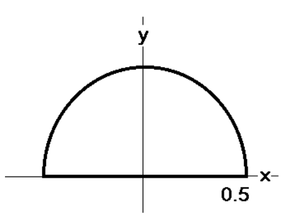
If the magnetic field is in the same direction as the velocity, no Lorentz force is exerted so the particle will travel with a constant velocity.

(2) A wire can be viewed as a path along which charge carriers flow. You can write the following: in the Lorentz Force Law: $q\vec{v}\times I\vec{L}$ where $\vec{L}$ represents the vector length of the wire, and I is the *conventional* current. Now, an important thing to remember is this: if **L** represents a **closed** loop of some arbitrary shape (but, planar), then the net force on the wire loop will be zero if B is uniform.

(a) Suppose a long wire is running along the x-axis and the wire has a length of 1 m. You may assume the wire is symmetrically placed so that it runs from x=-0.5m to x=+0.5m. Furthermore, let the wire carry a current of 1 A towards the positive direction. Let us place this wire in a uniform magnetic field (B) directed along the +Y axis. What is the magnitude and direction of the force on the wire if B=1T?
Solution:

$\vec{F}=I\vec{L}\times\vec{B}=ILB(\hat{x}\times\hat{y})=ILB\hat{z}=(1A)(1m)(1T)\hat{z}=1\hat{z}N$

Notice that a wire of 1 m, with a current of 1 A perpendicular to a magnetic field of 1 T produces a force of 1N.

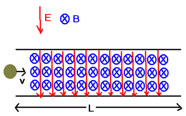


(b) Now suppose that your wire comes back with a semi-circular loop as shown below. What is the force on the semicircular loop?

Since the net force on the closed loop is zero, the net force on the semicircular loop is $\vec{F}=-1\hat{z}N$

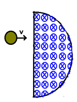
Note: I am going to, for now, use our definition of current in order to provide the definition for the magnetic field. Later, in more advanced study, in fact the current is defined in terms of the force exerted on it by the magnetic field and our older definition of current as the time rate of change of charge will kind-of go by the wayside.

(3) Motion of a charged particle in a combined electric and magnetic field. Explain how a *velocity selector* and a Mass spectrometer work.



The velocity selector is used to provide a beam of particles with a uniform velocity. A diagram of this is in the notes. Let the particles have velocities along the +x direction, and the magnetic field be directed in the +y direction and the electric field be directed in the -z direction. Then: $\vec{F}=q\vec{E}+q\vec{v}\times\vec{B}=-qE\hat{z}+qvB\hat{z}$. The net force is zero when:

$-qE+qvB=0\Rightarrow v=\frac{E}{B}$. Particles with velocities other than this will be deflected upward or downward. Thus a beam of charged particles of uniform velocity emerge from the velocity selector.



Now allow these particles to be inserted into a region of space where the magnetic field is given by: $\vec{B}=B_1\hat{y}$. The particles will undergo uniform circular motion (ignoring radiation effects). The force on the particle will be given by: $\vec{F}=q\vec{v}\times\vec{B}_1=q\left[\frac{E}{B}\right]\hat{x}\times B_1\hat{y}=q\left[\frac{E}{B}\right]\hat{x}\times B_1\hat{y}=q\left[\frac{E}{B}\right]B_1\hat{z}$. The particle will undergo a centripetal acceleration given by $a_c=\frac{v^2}{R}$. We can obtain the radius of orbit then:

$F=ma\Rightarrow qvB_1=m\frac{v^2}{R}\Rightarrow R=\frac{mv}{q}B_1=m\frac{\left[\frac{E}{B}\right]}{qB_1}$. If each of the particles are charged the same, then this will separate out the part of the beam according to their masses. Detectors are then placed along the walls and the particles could be detected electrically. One final detail: if the distance between the velocity selector plates is d, in terms of the potential difference developed across the plates, the radius is given by:

$$R=\frac{mv}{qB_1}=m\frac{[\frac{E}{B}]}{qB_1}=m\frac{V}{qdBB_1} .$$

(4) A singly charged positive ion has a mass of $3.2\text{x}10^{-26}$ kg. After being accelerated through a potential difference of 833 V, the ion enters a magnetic field of 0.92T along a direction perpendicular to the direction of the electric field. Calculate the radius of the path of the ion in the magnetic field.

2016	Physics **250**: Worksheet 11	Name ______________

Note: in this solution, I am using u to represent velocity to avoid confusion with the electric potential, V. When an ion is accelerated through a potential difference, it acquires a kinetic energy given by:

$$\frac{1}{2}mu^2 = eV \Rightarrow u = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.6x10^{-19})(833)}{3.2\,x\,10^{-26}\,kg}} = 9.13x10^4\frac{m}{s}$$

The Lorentz force then gives us the magnetic force on the ion:

$$F = quB = m\frac{u^2}{R} \Rightarrow R = \frac{mu}{qB} = \frac{(3.2x10^{-26})(9.13x10^4)}{(1.6x10^{-19})(0.92)} = 0.0198\,m$$

The fact that eV is indeed energy directs us to a useful unit you'll often see used in high energy physics, the electron volt. Basically 1 eV is $1.6x10^{-19}$J.

(5) A current I=15A is directed along the positive x-axis in a wire perpendicular to a magnetic field. The current experiences a magnetic ***force per unit length*** of 0.63 N/m in the negative y direction. Calculate the magnitude and direction of the magnetic field in the region through which the current passes.

Solution:

The magnetic force is given by $\vec{F} = I\vec{L}\times\vec{B}$, even though we're later going to talk about force per unit length. Now, we're told that the direction for the current is in the +x direction. We're also told that the force on the wire is in the –y direction. We thus need to find the correct cross product that will give this.

All the cross products are given by:

$$\hat{x}\times\hat{y} = \hat{z} \quad \hat{y}\times\hat{x} = -\hat{z} \quad \hat{x}\times\hat{x} = 0$$
$$\hat{y}\times\hat{z} = \hat{z} \quad \hat{z}\times\hat{y} = -\hat{x} \quad \hat{y}\times\hat{y} = 0$$
$$\hat{z}\times\hat{x} = \hat{y} \quad \hat{x}\times\hat{z} = -\hat{y} \quad \hat{z}\times\hat{z} = 0$$

To find the direction, it's just a matter of fitting up $\hat{x}\times[?] = -\hat{y}$.

You can see pretty quickly the unknown direction is the +z direction.

We can also find the magnitude of B now:

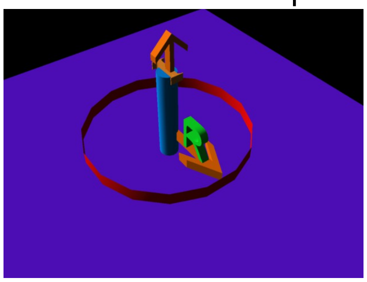
$$\frac{F}{L} = IB \Rightarrow B = \frac{1}{I}\left(\frac{F}{L}\right) = \frac{0.63}{15} = 0.042\,T$$

The required magnetic field is then: $\vec{B} = 0.042\,T\,\hat{z}$

2016	Physics **250**: Worksheet 11	Name _______________

(6) A wire of radius a carries a current I. Find the magnetic field produced at distances R outside the wire.

The fact that currents produce magnetic fields was discovered by Oersted during a demonstration for his class (and perhaps not completely by chance). He observed the deflection of a compass needle in the presence of a current. This forms the basis for our calculation of magnetic fields from current distributions. The mathematical formulation of this observation was made by Ampere in what we now know as Ampere's law. Ampere's law is valid for steady currents but it will need modification in the event that electric fields show a time dependence.



The mathematical statement of Ampere's law is (for s a closed path, and I_c the current cutting through the area bounded by the closed path):

$$\oint_{\text{path}} \vec{B}\cdot d\vec{s} = \mu_0 I_c$$

It is easier for me to show you how to apply Ampere's law. In this case, choose a circular loop around the wire (this is the "Amperian Loop").
The current cutting through the path is simply I. The path is given by: $s = 2\pi R$ and on this path, B is uniform in direction and parallel to the path. The direction is given by a right hand rule. Thus, we have: $B[2\pi R] = \mu_i I \Rightarrow B = \frac{\mu_0 I}{2\pi R}$. The direction is around the wire.