r21	Physics **250**: Worksheet 15	Name: ________________

(1) Inductance: (a) Calculate the inductance of a uniformly wound solenoid with N turns and length w and cross sectional area A. Assume w is long compared to the radius and that the core of the solenoid is in air. (b) Suppose the solenoid had $A=4x10^{-4}$ m^2, N=300 and w=0.25m. What is the inductance in this case? (c) Now if you decrease I at the rate of 50 A/s, what will be the induced emf?

(2) Mutual Inductance: A long solenoid of length w has N_1 turns, carries a current I and has a cross-sectional area A. A second coil containing N_2 turns is wound around the center of the first coil. Find the mutual inductance of the system assuming perfect flux coupling. Provide a numerical result with correct SI units if $N_1=500$, $A=3x10^{-3}$ m^2 w=0.5 m, and $N_2=8$.

(3) Magnetostatic energy: Suppose the solenoid in problem 1 carries a current of 5 A. calculate the stored magnetostatic energy and the magnetostatic energy density in the solenoid.

(4) Show how transformers work. Suppose that the number of turns are $N_p=1000$ and $N_s=2500$. If an emf of $\mathscr{E}_p$ =10 V is applied, what is the output voltage. If the turns are $N_p=2500$ and $N_s=1000$, what is the output voltage?

(5) How does the transformer transform current? What about a dc source?

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Solution:

We use the definition of inductance to calculate this: $L=\frac{\Phi_M}{I}$.

The magnetic field inside a solenoid is given by $B=\mu_0 n I$. Thus the magnetic flux through 1 single turn is given by $\Phi_{M,1}=BA=\mu_0 n I A$

The solenoid has N such turns, each of area A. Thus the total magnetic flux will be:
$\Phi_M=N\Phi_{M,1}=N\mu_0 nIA=\mu_0 n^2[Aw]=\mu_0 n^2 I[\text{volume}]$. We can now find the inductance:

(a) $L=\frac{\Phi_M}{I}=\mu_0 n^2[\text{volume}]$

(b) In the present case, we have: $L=[4\pi x10^{-7}]\left[\frac{300}{25}\right]^2[4x10^{-4}x0.25]=1.81x10^{-4}H$

(c) The emf is given by Faraday's law: (using $LI=\Phi_M$) as

$$\mathcal{E}=-\frac{d\Phi_M}{dt}=-\left[\frac{d(LI)}{dt}\right]=-L\frac{dI}{dt}=-1.81x10^{-4}[-50]=9.05x10^{-3}V$$

The moral here is this:
Be very careful how you turn off your magnets! It is easy to destroy controlling circuitry!

Notice that measurements of flux are not the most convenient way to measure this "self inductance." Indeed, it is much more convenient to use Faraday's law to measure the inductance. Since $\mathcal{E}=-\frac{d\Phi_M}{dt}$ and $\Phi_M=LI$ we have that the emf induced is given by $\mathcal{E}=-L\frac{dI}{dt}\Rightarrow L=-\frac{\mathcal{E}}{\left(\frac{dI}{dt}\right)}$. The procedure is this: apply a current which varies with time linearly, thus giving a good measure of the rate of change of the current. Then measure the emf produced. Divide the two to obtain the inductance of the circuit. For example: suppose the current is increasing at 5 A/s and an emf was measured to be -5V. The inductance would then be 1H, carefully avoiding problems with the minus sign.

(2) Mutual Inductance: A long solenoid of length w has N_1 turns, carries a current I and has a cross-sectional area A. A second coil containing N_2 turns is wound around the center of the first coil. Find the mutual inductance of the system assuming perfect flux coupling. Provide a numerical result with correct SI units if N_1=500, A=3x10^{-3} m^2 w=0.5 m, and N_2=8.

Solution: the mutual inductance is defined very much like the inductance of a system. In this case, however, the magnetic flux from one coil couples to the second coil to induce the emf. The mutual inductance is defined by: $M_{21} \equiv \frac{\Phi_{M_{2,1}}}{I_1}$, where my unusual flux symbol indicates that the flux in coil 2 is from the magnetic field produced from coil 1. We can find this flux, assuming perfect flux coupling. The magnetic field in coil 1 is given by: $B_1 = \mu_0 n_1 I_1$, so that the flux would be

$$\Phi_{M_{21}} = \mu_0 n_1 I_1 [N_2 A] = \mu_0 n_1 n_2 [\text{volume}] \ .$$

We can thus find the mutual inductance as: $M_{21} = \mu_0 n_1 n_2 [\text{volume}]$
In the numerical case above, we have:

$$M_{21} = [4\pi x 10^{-7}] \left[\frac{500}{0.5}\right] \left[\frac{8}{0.5}\right] [3 x 10^{-3} x 0.5] 3.02 x 10^{-5} H = 30.2 \mu H$$

Although M_{12} and M_{21} appear to be different, normally they are not and are simply referred to as M (the mutual inductance).

(3) Magnetostatic energy: Suppose the solenoid in problem 1 carries a current of 5 A. calculate the stored magnetostatic energy and the magnetostatic energy density in the solenoid.

The work required to establish a current I in an inductor is given by the magnetostatic energy. Probably this is most easily obtained by looking at Faraday's law:

$$\mathcal{E}=\frac{-d\Phi_M}{dt}=-L\frac{dI}{dt} \quad .$$

The power required to establish this emf in the presence of an existing current is given by:

$$P=\mathcal{E}I=LI\frac{dI}{dt} \quad .$$

From this, we can find the stored energy:

$$U_M=\int_{t=0}^{t} P\,dt=\int_{I=0}^{I} LI\,dI=\frac{1}{2}LI^2 \quad .$$

A non-calculus method of doing this is to look at the same expression:

$$\mathcal{E}=-L\frac{\Delta I}{\Delta t}\Rightarrow \Delta U_M=\mathcal{E}I\Delta t=LI\Delta I \quad .$$

From our definition of inductance, we know that the flux is given by $\Phi_M=LI$ so that we have

$$\Delta U_M=\Phi_M\Delta I$$

and thus, the average value of flux that the current must change is given by

$$\frac{1}{2}\Phi_M=\frac{1}{2}LI \quad .$$

In this time, Δt, I varies from 0 to I. Thus the change in stored magnetostatic energy is given by

$$\Delta U_M=\frac{1}{2}LI^2$$

or more simply

$$U_M=\frac{1}{2}LI^2 \quad .$$

I do find the non-calculus version of this to be somewhat displeasing and really like the calculus version more. Now you are really going to ultimately want to write this in terms of the magnetic field. Since the magnetic field is given by

$$B=\mu_0 nI\Rightarrow I=\frac{B}{\mu_0 n}$$

and the inductance is given by

$$L=\mu_0 n^2[\text{volume}] \quad ,$$

we can calculate the magnetostatic energy as

$$U_M=\frac{1}{2}[\mu_0 n^2(\text{volume})]\left[\frac{B}{\mu_0 n}\right]^2=\frac{1}{2\mu_0}B^2[\text{volume}] \quad .$$

It's easy to see from this that the magnetostatic energy density is

$$u_M \equiv \frac{U_M}{[\text{volume}]} = \frac{1}{2\mu_0} B^2 \quad .$$

To answer the previous problem, we already have the inductance of the solenoid: $L = 1.81 \text{x} 10^{-4} \text{H}$ so the energy is

$$U_M = \frac{1}{2}[1.81 \text{x} 10^{-4}][5]^2 = 2.25 \text{x} 10^{-3} \text{ J} \quad .$$

The energy density is then given by

$$u_M = \frac{2.25 \text{x} 10^{-3} \text{J}}{(4 \text{x} 10^{-4} \text{x} 0.25) \text{m}^3} = 22.5 \frac{\text{J}}{\text{m}^3} \quad .$$

An important note: the magnetostatic energy density is as important to magnetic systems as the electrostatic energy density, $u_E = \frac{1}{2} \epsilon_0 E^2$ was to electrostatic systems. The magnetostatic energy density is given by $u_M = \frac{1}{2\mu_0} B^2$. You will see this again this semester, as you will also see the electrostatic energy density again this semester.

(4) Show how transformers work. Suppose that the number of turns are N_p=1000 and N_s=2500. If an emf of $\mathcal{E}_p$=10V is applied, what is the output voltage. If the turns are N_p=2500 and N_s=1000, what is the output voltage?

A transformer consists of two coils of wire which are strongly coupled (inductively) by means of an iron core. Almost all the magnetic flux lines from the first coil pass through the second coil, and the presence of the iron core enhances this effect. Suppose that coil 1 has N_p (primary side) turns while coil 2 has N_s (secondary side) turns. Now suppose that the primary side has an applied voltage which is an AC voltage. Find out how the voltage output behaves.

Solution: By Faraday's law and the discussion in problem (2) we have the following case:

$$\mathcal{E}_1 = -N_1 \frac{d\Phi_M}{dt} \text{ and } \mathcal{E}_2 = -N_2 \frac{d\Phi_M}{dt} \quad \text{(calculus version)}$$

For the transformer, almost all the flux lines passing through the first coil pass through the second coil (we assume perfect coupling. This is enforced by placing iron in the transformer). Thus we have

$$\frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{N_1}{N_2} \Rightarrow \mathcal{E}_2 = \mathcal{E}_1 \left[\frac{N_2}{N_1}\right]$$

In terms of the subscripts “p” for primary and “s” for secondary we have the transformer equation:

$$\mathcal{E}_s = \mathcal{E}_p \left[\frac{N_s}{N_p}\right]$$

For the problem at hand, in the first case the output voltage is 25 V while in the second case, the output voltage is 4V.

(5) How does the transformer transform current? What about a dc source?
Solution: Here I’ll assume the transformer is almost 100% efficient meaning that the power input to the transformer is equal to the power output. Since P=IV we then have the result $P_p = P_s$ .But the power is given by: $\mathcal{E} I = P$. We thus have the result:

$$\mathcal{E}_p I_p = \mathcal{E}_s I_s \Rightarrow I_s = \frac{\mathcal{E}_p}{\mathcal{E}_s} I_p \quad .$$

We can use the transformer equation now to obtain the result: $I_s = I_p \frac{N_p}{N_s}$ which says that transformers work exactly the opposite way for currents as opposed to voltages. Thus a transformer which increases voltage (a step-up transformer) decreases current and a transformer which decreases the voltage (a step-down transformer) increases the current.

If you try to transform a DC source like the output from a battery, you will be generally disappointed with the results since in order for a transformer to work, you need a time rate of change of magnetic flux, which you won't have with a dc source! However, sometimes using a transformer to remove a DC offset is desired and at these times, a transformer does exactly what you want it to do.