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(1) A proton $m_p=1.67x10^{-27}$kg is moving with a velocity v=0.01c (c is the speed of light, $c=3x10^8$ m/s) at right angles to a magnetic field which is 10 T in strength. What is the radius of the orbit of the proton. (ignore radiation effects here). The charge on the proton is $q=+1.602x10^{-19}$C.

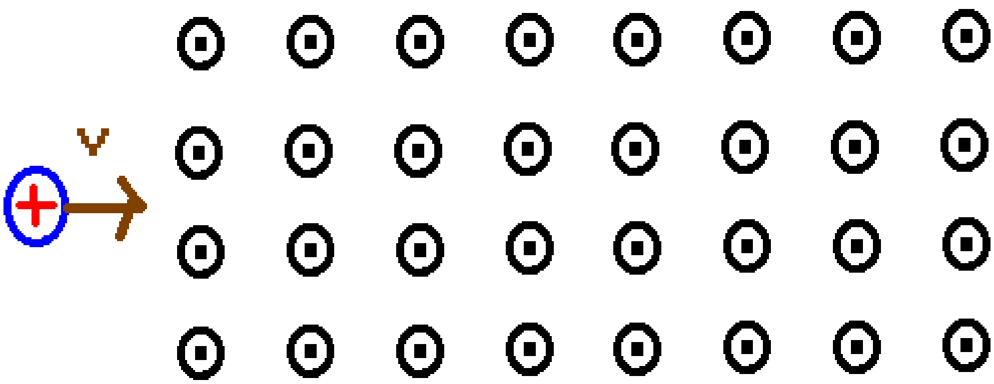
(2) A wire has a length of 0.78 m and carries a current of 0.35 A in the +y direction. The wire experiences an upward force of 0.10 N. What is the direction and the strength of the magnetic field which is at right angles to this wire?

(3) Show that a closed loop carrying a current I in a uniform magnetic field experiences no net external force.

The fundamental observation one makes regarding a magnetic field is that a moving electron can be directed so that it is deviated from a straight-line path when it encounters a magnetic field.

The SI unit of magnetic field is a Tesla (T)
I'll show you later how you can measure and define this unit.
The problem of magnetic fields is a 3-dimensional problem. We need to be able to represent the magnetic field vectors coming into or out of the plane of a surface.
Unicode characters: u2299 = ⊙ and u2295=⊕ and u2297=⊗
⊗ represents a vector going into the screen
⊙ represents a vector coming out of the screen.

The observation:



A charged particle moving at right angles to a magnetic field will be deviated by an apparent force which acts at right angles to the instantaneous velocity.

The other two possibilities can be seen by "going behind" the plane of the screen.

We can represent this force by use of the cross product.

For a charge q in motion, the vector "magnetic force" on the particle is
Note: the unicode symbol for × is "control shift u 2a2f"

$$\vec{F} = q\vec{v} \times \vec{B}$$

where $\vec{v}$ is the vector velocity and $\vec{B}$ is the magnetic field in the region of the particle.
This force is called the Lorentz force.
It is important to note that only components of the magnetic field which are perpendicular to the velocity produce this force. Parallel components will have no effect.

Let's do an example:
Suppose a particle (Q=+1 C) has a velocity given by: $v = v_0 \hat{x}$ and it experiences a uniform magnetic field given by $\vec{B} = B_0 \hat{y}$. What is the force acting on the particle?

The cross product is defined by: $\hat{x} \times \hat{y} = \hat{z}$ and $\hat{y} \times \hat{x} = -\hat{z}$.

Here, then we have: $\vec{F} = q v_0 B (\hat{x} \times \hat{y}) = q v_0 B \hat{z}$.

You will want to sketch a picture of what happens. In short, the force will always be perpendicular to this velocity and this type of force produces uniform circular motion. From Newton's law, we can calculate the radius of orbit for the particle:

$$F = m \frac{v^2}{R}$$

for a particle undergoing uniform circular motion. We then have from the fundamental observation the fact that $|\vec{F}| = q v B$.

Thus, the radius of orbit is given by: $qvB = m \frac{v^2}{R} \Rightarrow R = m \frac{v}{qB}$.

In a more advanced study of electricity and magnetism, you would find that when a charge undergoes an acceleration, power must radiate from the charge. This particle would not go in a circular path forever, therefore. However, we won't pursue this aspect further in our course.

What happens to current in the presence of a magnetic field?

Now, imagine positive charges constrained to move along a straight line in the presence of a magnetic field. If you like, you can say that a wire can cause this to happen. Incidentally, I would imagine that for a high enough current in a wire which is placed at right angles to a magnetic field, there might be some possibility to eject moving electrons from the surface of the wire, but I am not aware of any experiments which report this.

What will happen is this: the positive charges (moving) in the wire will experience a force due to the magnetic field.

We don't usually (i.e. ever) actually see protons moving so actually the fundamental observation probably ought to be the observation of what happens to a current in the presence of a magnetic field.

Here is a second subtle point: although a wire might have a current in it and although we did define the current in terms of the motion of charged particles, in reality the particles in the wire are not moving very far. How a current is transferred through the wire is actually due to a slight motion of electrons at one end of the wire transmitting the electric force through the wire.

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Suppose a wire is carrying a current (the conventional current) in the +x direction in the presence of a magnetic field which is in the +y direction. According to our fundamental observation, the moving charges in the wire will experience a magnetic force which is perpendicular to both the magnetic field and the velocity. Let's look at the mathematics of this problem:

We've previously written: $I=\frac{\Delta q}{\Delta t}$.

Now, we're going to put a vector onto the current and relate it to the velocity of a charged particle.

Let's assume that the particles are moving with a constant velocity: $\vec{v}=v_0\hat{x}$.

In a time t, the particles will move through a distance given by: $(\Delta x)\hat{x}=v_0 t\hat{x}$.
We can then imagine a column of such particles which has a cross sectional area A.

The volume of such particles moving past a given point in space during a time t is: $\text{volume}=v_0 A t$.

If the charges in this volume have a volume charge density ρ, then the total charge moving is:

$$Q=\rho v_0 A t \quad .$$

The current associated with this motion of charge is then:

$$I=\frac{\Delta Q}{\Delta t}=\rho v_0 A \quad .$$

Now, let's concentrate on a line segment of length L.
(note: be careful, later on we're going to also use L to represent inductance).

Then, we have:

$$IL=\frac{\Delta Q}{\Delta t}L=\rho v_0 AL=q v_0 \quad .$$

What I have shown you now is that we can either deal with charges moving or lengths of current.

We're going to want to put a vector sign on this. We're going to call the conventional current the direction that positive particles are moving. However, here, we'll let the vector be carried by the wire length. Thus, in this example:

$$I\vec{L}=IL\hat{x}$$

The force that a current would experience then is given by:

$$\vec{F}=q\vec{v}\times\vec{B}=I\vec{L}\times\vec{B} \quad .$$

We define the direction of L to be that direction in which positive charges are moving.

Here is another example now:
Suppose a magnetic field of 1 T is directed along the +y direction and a wire 0.1 m long carries a current of 1.0 A. What **vector** force will the wire experience?

The force on the wire is given by:

$$\vec{F} = I\vec{L} \times \vec{B} \quad .$$

In this case, we then have:

$$\vec{F} = ILB(\hat{x} \times \hat{y}) \quad .$$

We thus find:

$$\vec{F} = ILB\,\hat{z} = (1.0\text{A})(0.1\text{m})(1.0\text{T})\,\hat{z} = 0.1\,\text{N}\,\hat{z} \quad .$$

Now, you've probably wondered about those magnetic units of field called Tesla. Let's step back and explore this.

Suppose you have a wire with a length of 1.0 m and the wire carries a current of 1.0A. We'll run the wire through a magnetic field (at right angles) and measure the force on the wire. The result is this:

I: Define the magnetic field direction

(a) let the conventional current be directed in the +x direction

(b) position the magnet so that the force on the wire is observed to be in the +z direction

(c) the direction of the magnetic field is in the +y direction.

II: Define the strength of the magnetic field

(a) Increase the magnetic field strength until 1 N of force is measured.

(later, you'll see how one increases magnetic field strength)

If the conventional current is in the +x direction and is equal to 1A, and if the force is in the +z direction and is equal to 1N, then the magnetic field is in the +y direction and is equal to 1 T.

In more familiar terms, the Tesla can be seen to have units of:

$$[B] = \frac{\text{N}}{\text{A m}} \quad .$$

Now, it's going to turn out that the definition for the Tesla which I have just given you is, in fact, not how a Tesla is defined. Later, we'll be able to calculate magnetic field strengths directly from currents. The force on a wire will then be used to provide a definition for current instead of magnetic field strength.

Do be aware, however, that at this point, I am not telling you the whole story about magnetic fields!

In the lab, you will be measuring directly the force on a current carrying wire in order to determine the magnetic field strength of a permanent magnet.

Now, what happens if the wire carrying the current is not at right angles to an external magnetic field?

The force on a current-carrying conductor is given by:

$$\vec{F} = I\vec{L} \times \vec{B} \quad .$$

The force will always be perpendicular to both L and B. However, what if L and B are not at right angles?

We have a way to write the magnitude of the force in terms of the cross product of two vectors. Recall, that for 2 vectors,

$$|\vec{C} \times \vec{D}| = |\vec{C}||\vec{D}||\sin(\theta)|$$

where θ is the angle between C and D. You might as well refer this to the smallest angle between these two vectors.

If the wire is not at right angles to the magnetic field, you will observe the following:

$$|\vec{F}| = ILB\sin(\theta) \quad .$$

In the lab, you will also be able to verify this angular dependence for the force on a current-carrying conductor.

(1) A proton $m_p=1.67x10^{-27}kg$ is moving with a velocity v=0.01c (c is the speed of light, $c=3x10^8$ m/s) at right angles to a magnetic field which is 10 T in strength. What is the radius of the orbit of the proton. (ignore radiation effects here). The charge on the proton is $q=+1.602x10^{-19}C$.

Solution:

$$\vec{F}=q\vec{v}\times\vec{B}\Rightarrow|\vec{F}|=qvB$$

This force is always at right angles to the velocity and is thus a central force. So:

$$F=m\frac{v^2}{R} \quad .$$

We can find the radius by equating the two forces:

$$m\frac{v^2}{R}=qvB\Rightarrow R=\frac{mv}{qB}=\frac{(1.67x10^{-27}\,Kg)(0.01x3x10^8\frac{m}{s})}{(1.602x10^{-19}C)(10\,T)}=3.13x10^{-3}\,m$$

If you want a tighter orbit, you can increase the magnetic field or increase the particle velocity. However, do be aware that as the orbit increases, so does the acceleration of the particle which will result in increased radiation from the particle and hence, energy loss.

(2) A wire has a length of 0.78 m and carries a current of 0.35 A in the +y direction. The wire experiences an upward force of 0.10 N. What is the direction and the strength of the magnetic field which is at right angles to this wire?

Solution:
It is easy to find the magnetic field strength from F=ILB. The field strength is then given by:

$$B=\frac{F}{IL}=\frac{0.1\,N}{(0.35\,A)(0.78\,m)}=0.366\,T$$

The direction needs to be looked at carefully: for the cross product, we have:

$$\hat{x}\times\hat{y}=\hat{x} \qquad \hat{y}\times\hat{z}=\hat{x} \qquad \hat{z}\times\hat{x}=\hat{y}$$
$$\hat{y}\times\hat{x}=-\hat{z} \qquad \hat{z}\times\hat{y}=-\hat{x} \qquad \hat{x}\times\hat{z}=-\hat{y}$$

(it's easy to remember (x,y,z) (y,z,x),(z,x,y) give positive cross products)
Now, we defined the +y direction to be that direction in which the current is flowing. Let's pick the +z direction to be upward. Then, the direction of B that would produce a force in the +z direction is, in fact, going to have to be in the –x direction.
Thus: $\vec{B}=-0.366\,T\,\hat{x}$

(3) Show that a closed loop carrying a current I in a uniform magnetic field experiences no net external force.

This problem is one subtle puppy that is often not even mentioned in text books. In fact, you really need to search the text books to find this. I used to be a bit uneasy talking about this until I did finally find it in a text book.
first:
What if you have several segments of wire, each of which carry a current I in a uniform magnetic field. What is the total force on the segments?
the force on any one segment is given by:

$$\vec{F} = I\vec{L} \times \vec{B}$$

We need, however, to superimpose the forces of several segments. Thus:

$$\vec{F} = \sum_{i\ \text{segments}} I\vec{L}_i \times \vec{B}$$

Ok, I'm going to rewrite this slightly …

$$\vec{F} = -\sum_{i\ \text{segments}} \vec{B} \times I\vec{L}_i = -I\vec{B} \times \sum_{i\ \text{segments}} \vec{L}_i$$

I don't think you'll have any mathematical problem with this … as an example, consider two segments, L1 and L2. Then,

$$\vec{F} = -\sum_{i\ \text{segments}} \vec{B} \times I\vec{L}_i = -I\vec{B} \times \left(\vec{L}_1 + \vec{L}_2\right) = -I\vec{B} \times \sum_{i\ \text{segments}} \vec{L}_i$$

at least, that's how I convinced myself that it was ok to do what I've just done. Now remember, this only works if B is uniform at each segment. Otherwise, all bets are off.
Now, you need to ask yourself this: what is that sum of segments? Remember about the 2nd week of 1st semester physics when I introduced you to vectors? We there defined the displacement vector as the vector sum of all individual vectors that made up a trip. I probably even told you to remember that for later use.

$$\vec{D} \equiv \text{displacement} = \sum_{i\ \text{segments}} \vec{L}_i$$

And, $\vec{D} = \vec{0}$ for a closed loop.

In particular, we want to take a round trip.
If we do, then the displacement vector is zero.

Thus, for a closed loop:

$$\sum_{i\ \text{segments}} \vec{L}_i = \vec{0} \Rightarrow \vec{F}_{\substack{\text{magnetic} \\ \text{closed loop}}} = \vec{0}$$

Now you might ask yourself … what good is this? Let me show you.

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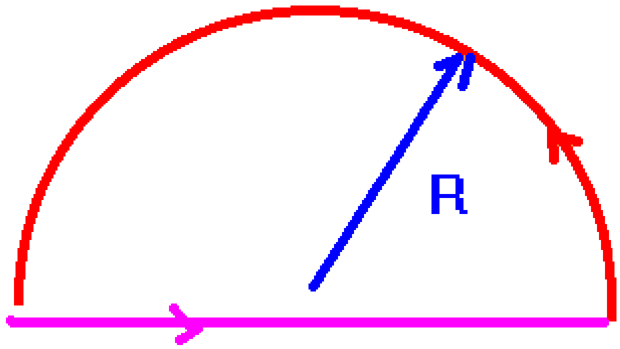
The circular loop shown carries a current I. A uniform magnetic field is directed into the screen. Find the net magnetic force on this portion of the wire.



It’s not too easy to add up all those little segments of the wire. In fact, really only the calculus students can do it directly through:

$$\vec{F} = \oint_{\theta=0}^{\theta=\pi} IR\,d\theta\left(\hat{\theta} \times \vec{B}\right)$$

On the other hand, you know my little theorem above which says that the net magnetic force on a closed loop is exactly zero if the loop is in a uniform magnetic field.

Consider the purple line carrying the current in the direction shown. When it joins up, we have a closed loop and the net magnetic force is equal to zero. You can easily calculate the magnetic force on this wire since it is equal to:

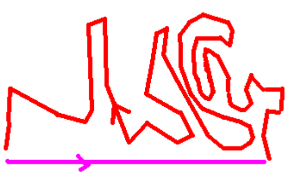
$$\vec{F} = 2IRB\left(\hat{x} \times \hat{y}\right) = 2IRB\,\hat{z}$$

where I’ve let the right hand direction be +x and into the screen be +y. +z is the upward direction.

Since the total force on the closed loop must be zero, we then have:

$$\vec{F} = -2IRB\,\hat{z}$$

on the circular loop.



In fact, the red loop could look like this and the same result will hold. I think, however, for application of this result, you need to be careful about letting part of the circuit go to infinity and then come back. You might have problems then. I’m also a bit uneasy about a “fractal path” here but it ought to be ok, I believe.