Different Versions of Biot Savart Law r2018

Note: Strict application is only to steady currents

(a) A line of current directed along a path length L

$$\vec{B}(\vec{r}_p)=\frac{\mu_i I}{4\pi}\int_{\text{all currents}}\frac{d\vec{L}_i \times \vec{r}_{ip}}{|\vec{r}_{ip}{}^3|}$$

(b) From a surface current:

A surface current would be defined by:

$$\vec{I}=\int_{\text{surface}}\vec{K}(\vec{r}_i)\,dA$$

or perhaps by

$$\vec{K}=\frac{d\vec{I}}{dL_\perp}$$

Or by:

$$\vec{K}=\sigma\vec{v}$$

for σ being a surface charge density.

In words, "K is current per unit width (L^1) perpendicular to flow"

Note that the units of K are A/m.

$$\vec{B}(\vec{r}_p)=\frac{\mu_0}{4\pi}\int_{\text{all currents}}\frac{\vec{K}(\vec{r}_i)\times\vec{r}_{ip}}{|\vec{r}_{ip}|^3}\,dA_i$$

(c) From a volume viewpoint

(note that the symbol is J which in other courses has a different meaning)

In words, "J is current per unit area (L^2) perpendicular to the flow"

$\vec{J}=\rho\vec{v}$ for ρ being a volume charge density.

Note that the units of J are A/m^2.

With:

$$\vec{B}(\vec{r}_p)=\frac{\mu_0}{4\pi}\int_{\text{all currents}}\frac{\vec{J}(\vec{r}_i)\times\vec{r}_{ip}}{|\vec{r}_{ip}|^3}\,d\tau_i$$

The last definition gives us Ampere's law:

$$\int(\vec{\nabla}\times\vec{B})\cdot d\vec{A}=\oint\vec{B}\cdot d\vec{L}=\mu_0\oint\vec{J}\cdot d\vec{A}\Rightarrow\oint\vec{B}\cdot d\vec{L}=\mu_0 I_c$$

by using the differential form of Ampere's law which is

$$\vec{\nabla}\times\vec{B}=\mu_0\vec{J}$$