B from a wire of length 2a 2016

A wire of length 2a carries a current I along the +x axis. Find the magnetic field along the +z symmetry axis.

The law of Biot-Savert says:

$$\vec{B}=\frac{\mu_0 I}{4\pi}\int\frac{d\vec{L}_i x\,\vec{r}_{ip}}{r_{ip}^3}$$

$$d\vec{L}_i=dx\,\hat{x}:\vec{r}_i=x\,\hat{x}:\vec{r}_p=z_p\,\hat{z}:\vec{r}_{ip}=-x\,\hat{x}+z_p\,\hat{z}:d\vec{L}\,x\,\vec{r}_{ip}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ dx & 0 & 0\\ -x & 0 & z_p\end{vmatrix}=-dx\,z_p\,\hat{y}$$

$$\vec{B}=-\hat{y}\frac{\mu_0 I}{4\pi}z_p\int_{x=-a}^{x=a}\frac{dx}{\left[x^2+z_p^2\right]^{3/2}}$$

From the alpha site:

int 1/((x ^2 + z ^ 2) ^ (3/2)) from -a to +a : $\displaystyle\int_{x=-a}^{x=a}\frac{dx}{\left[x^2+z_p^2\right]^{3/2}}=\frac{2a}{z_p^2\sqrt{a^2+z_p^2}}$

$$\vec{B}=-\hat{y}\frac{\mu_0 I}{4\pi}\frac{2a}{z_p\sqrt{a^2+z_p^2}}$$

Let's see if this reproduces the result from Ampere's law for an infinite wire:

$$\vec{B}=-\hat{y}\frac{\mu_0 I}{4\pi}\frac{2a}{z_p\sqrt{a^2+z_p^2}}=-\hat{y}\frac{\mu_0 I}{4\pi}\frac{2}{z_p\sqrt{1+\left(\frac{z_p}{a}\right)^2}}\approx-\hat{y}\frac{\mu_0 I}{2\pi z_p}$$

From Ampere's law, at a distance of z_p from an infinite wire, we have: $B=\dfrac{\mu_0 I}{2\pi z_p}$.

The direction is also correctly predicted and agrees with the right hand rule.