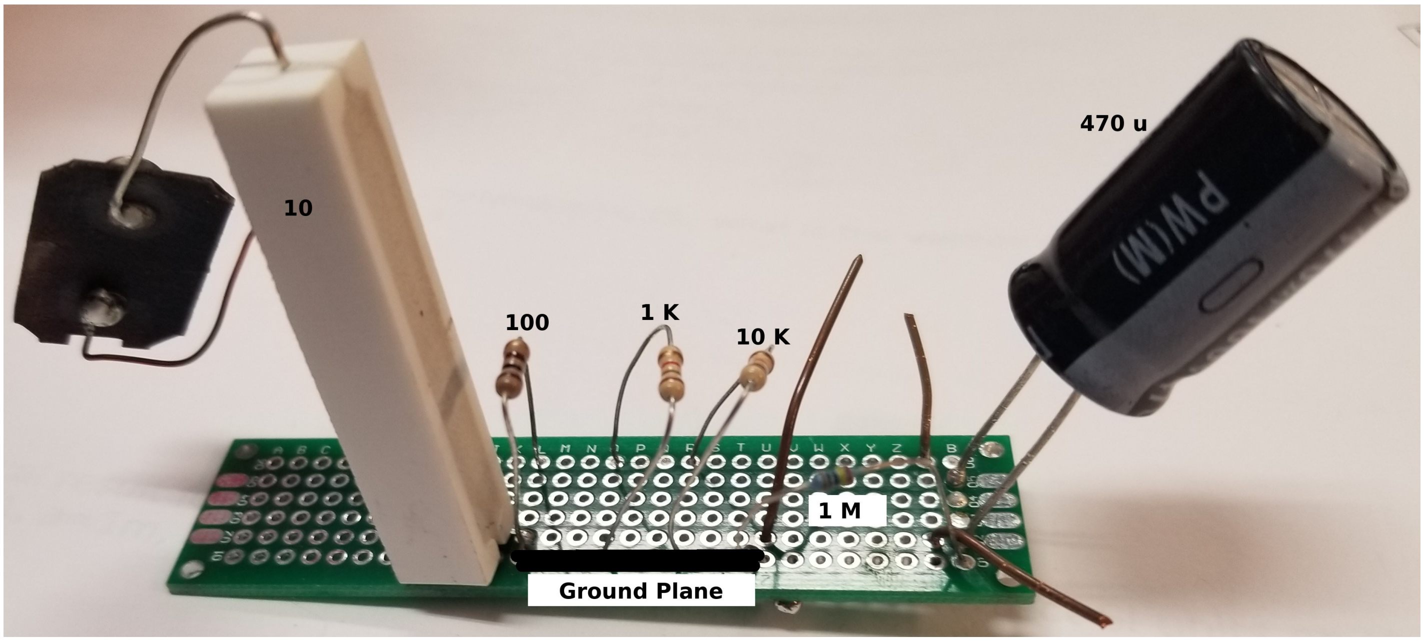
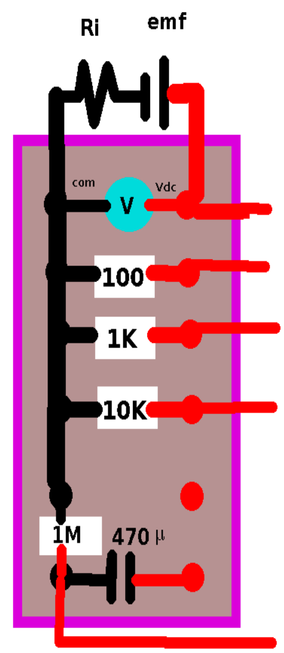
**Measurement of the EMF from a source,**
**Parallel and Series Capacitance and the RC time constant of a series RC circuit.**
***Revised for 2019-r3***

In this lab, you will explore aspects of the EMF from a battery and you will also investigate the time dependence of the series RC circuit and also parallel and series capacitance. The resistors you use here are color coded. The color coding can be found at many locations.

**Image of your circuit for today with elements marked**



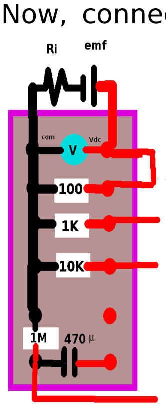




## Part 1: Measurement of the internal resistance of a battery.

The first part of this lab is intended to help you gain an understanding of the difference between an emf and an potential source. For this experiment, I have provided you with the circuit shown below. The emf source here is a battery which has some internal resistance, which I have added to the normal resistance from a battery. The idea is this: when you place your voltmeter across the open leads of a source of emf, if the voltmeter is ideal, you will not have a current through your circuit. In this configuration, you will measure the emf of your battery, but not the internal resistance that goes along with your battery. Of course, there may be some transient behavior that occurs so the voltmeter reading may slowly build up to a final steady reading for the emf.

You will want to record the values of R1, R2, R3, R and C for future reference at the beginning. Nominally, the values for the resistors are about R1=10, R2=100, R3=1K and R4=1Meg although variations may occur. Also the capacitor is about 470 $\mu$f . You ought to record these values for future reference at the beginning of lab.

Now, connect your circuit as shown to the right. This configuration allows you to measure the emf (or open-circuit voltage) from your battery. Make this measurement now and record it. It won't be too surprising that the result is around 7 volts (or less, as the batteries age). This is, then, the emf from your source.




All batteries have some internal resistance. In addition, I have placed an external resistance on the battery to add to this internal resistance. Let's see how to measure this and what effect it is going to have on your circuit.

EMF from the battery: _____________ volts

Now we are going to measure the internal resistance. This will be done by measurements of potential drops across the battery when various known resistances are placed in the circuit.

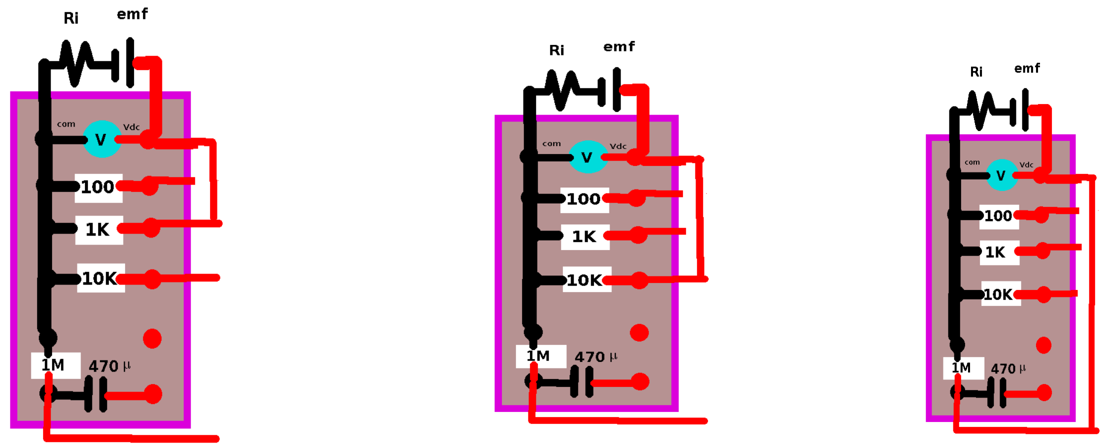
Make the additional connections shown in Figure 4 and observe what happens to the reading on the voltmeter when the final connection is made. **Don't leave this connected too long because it will drain the battery.**

**The terminal voltage will drop. Why?**

The internal resistance emf and terminal voltage (the terminal voltage is the voltage measured across the terminals of the battery) are related by:

$$V = \mathcal{E} - IR \quad .$$

When I is zero, you measure directly the emf of the battery. However, you will experience a potential drop due to the internal resistance of the battery once a current is established. Just how this allows a measurement of the internal resistance is worth looking at. Record the current and the terminal voltage for each of the other resistors, connecting them as shown in the images. The other connections are shown below.




Note: it is not necessary to fill in this table here on this page. You may record it in your notebook.

| Resistor [Ω] | Terminal Voltage [V] |
|---|---|
| 100 | |
| 1000 (1K) | |
| 10000 (10K) | |
| $1x10^6$ (1 M) | |

**Calculation of the internal resistance**

First determine the current through the circuit:

$$\mathcal{E} - IR_{internal} - IR_{external} = 0 \Rightarrow \mathcal{E} - IR_{internal} = IR_{external}$$

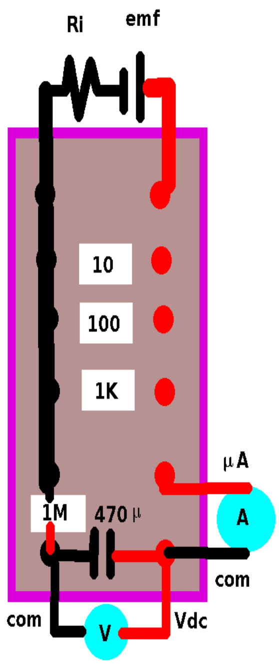
$$V_{terminal} \equiv \mathcal{E} - IR_{internal} = IR_{external} \Rightarrow I = \frac{(\mathcal{E} - V_{terminal})}{R_{external}}$$

Next, use this current to determine the internal resistance:

$$\mathcal{E} - IR_{internal} - IR_{external} = 0 \Rightarrow (\mathcal{E} - IR_{external}) = IR_{internal} \Rightarrow \frac{(\mathcal{E} - IR_{external})}{I} = R_{internal}$$

$$\Rightarrow R_{internal} = \frac{\mathcal{E}}{I} - R_{external}$$

Plot your values in the spreadsheet helper and provide a copy of the helper in your report today. You will probably see a very strong dependence upon current. Remember, the values on the plot are log-log so that a small change is actually quite large. Most correctly said: **the internal resistance is the resistance that is measured at zero current**. Large deviations in this plot are possible owing to small errors in measurement of currents so the most reliable measurement is at zero current.




## Part II: The series RC Circuit

### Connections for discharge/charge measurements from the RC circuit

You will want to begin by connecting the mx55 which configured as a voltmeter across the capacitor.

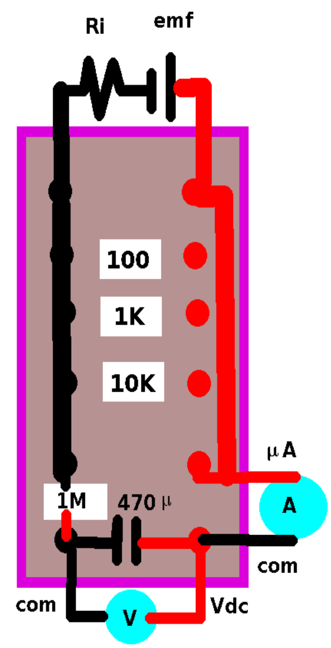
Then connect the com of the mx55 which is configured as an ammeter to the terminal on the unconnected side of the capacitor. The other terminal (µA) of the mx55 we will connect soon.

To quickly discharge the capacitor, touch a wire across the Vdc side of the voltmeter to the "com" side of the voltmeter, then remove this wire. This shorts out the capacitor.

With the capacitor completely discharged, you are now ready to start charging.

### Acquiring the charge data

Start your acquisition program and make the connection between the ammeter and the positive end of the battery as shown below.




In making these measurements, make sure your meters are using the 1000 MΩ input impedance, and, of course, the RS232 interface needs to be initialized.

Running of the data acquisition program should be straight forward. You'll name your file, press s to start and t to terminate. The computer will collect data about every 1/3 sec or so. After you press t for terminate, the program will write the data to the data file on the I drive. This may take a few minutes depending upon factors beyond my control. In any event, let the program close itself out ... don't close it out prematurely since you may not get all your data written properly. At the present time, the helper is limited to a maximum acquisition of 100 data tuples although the program can take 3500 tuples. Stop after about 200 seconds.

I suggest however, that when you cut and paste from the acquired data, select 100 measurements starting after about 15-20 seconds of running, discarding all other data. Paste this into the spreadsheet helper. Then look at and understand the analysis.

Next, you'll acquire data on the charging capacitor (write this to a different data file than the discharging system).

**Procedure for analyzing charge data**

Paste your data into the RC charge helper. You will want to use paste special, choose the option to detect special numbers, and paste it column by column, again starting after say 15-20 s of data to a maximum of 100 data tuples; in fact use exactly 100 data tuples.

The charging potential across the capacitor should vary as

$$V = \mathcal{E}\left(1 - e^{-t/\tau}\right) = \mathcal{E} - \mathcal{E}e^{-t/\tau}$$

While the charging current measured should vary as

$$I = \frac{\mathcal{E}}{R}e^{-t/\tau} \Rightarrow RI = \mathcal{E}e^{-t/\tau}$$

So, at each point throughout time, we can thus find the resistance and emf through time as:

$$V = \mathcal{E} - IR$$

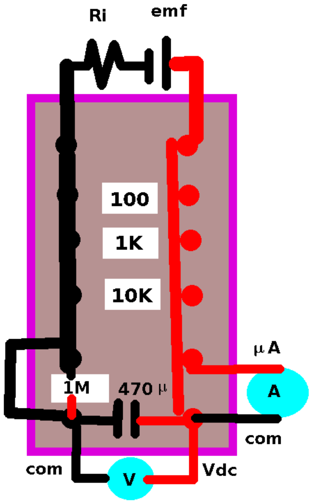
A plot of this will then give an intercept whose absolute value is the emf and the absolute value of the slope will be the resistance.

The calculation of this is provided on the helper for you by the slope and intercept functions.
Now that you have R, you can calculate the time constant and thus the capacitance from the discharge current. As before, look at the logarithm:

$$I = \frac{\mathcal{E}}{R}e^{-t/\tau} \Rightarrow \ln(I) = \ln\left(\frac{\mathcal{E}}{R}\right) - \frac{t}{\tau}$$

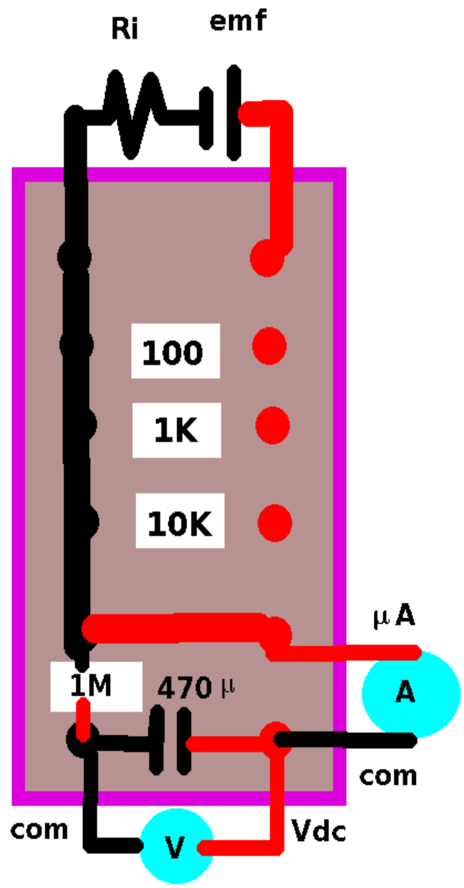
**The slope will provide the time constant from which the capacitance is calculated. This is provided also on the spreadsheet. You will want to compare the value of C obtained with the value of C which you measured earlier by the meter by using the %deviation. You will need to do this calculation by hand and report it in your report.**

**Acquiring discharge data**




In order to fully charge your capacitor quickly, look at the circuit connections shown below. You want to start with a fully charged capacitor since you are going to be measuring the discharge. Connect the positive side of the battery to the positive (V terminal) of the voltmeter and short across the resistor as indicated.

When you remove these two wires, you should have a fully charged capacitor. Now you are to start your acquisition program and make the final connection which I have shown below.



**Procedure for analyzing discharge data**

Paste your data into the RC discharge helper. You will want to use paste special, choose the option to detect special numbers, and paste it column by column**. It is not essential to paste beginning at t=0 and in fact, I don't recommend it. You might start after say t=10 s. However do paste 100 data tuples here total.**

The discharge potential across the capacitor should vary as

$$V=V_0 e^{-t/\tau}$$

While the discharge current measured should vary as

$$I=I_0 e^{-t/\tau}$$

**At each point** throughout time, we can thus find the resistance as:

$$R=\frac{V}{I}$$

which, of course, you know since this is the definition of resistance.

A plot of V vs I, together with the trend line is shown. Additionally the resistance calculation is provided by the openoffice slope function. Since the slope is linear, this means that the resistance is Ohmatic. Since the intercept is nearly zero, the implication is that the voltmeters supply little current to the circuit and also that the time dependence of V and I are identical.

We can measure the time constant by looking at the logarithm of the discharge current through:

$$\ln(I)=\ln(I_0)-\frac{t}{\tau}$$

A plot of this will show a slope with an absolute value of $\text{slope}=\frac{1}{\tau}$ .Since theoretically the time constant is RC, and since we have already obtained the resistance, the capacitance can now be determined as:

$$\frac{1}{\text{slope}}=RC \Rightarrow C=\frac{1}{R(\text{slope})} \quad .$$

You will want to compare the value of R obtained with the value of R which you measured earlier with the meter by using the %error:

$$\%\text{error}=100\left[\frac{(\text{measured}-\text{expected})}{\text{expected}}\right]$$

which the spreadsheet will do for you.

**Be sure to report the fit parameters in your report (emf, R, C, and RC). The results for the charging data are not perfect and you may observe significant variations in R and especially C. This would be due to the battery having additional electrical properties that we have not considered in this analysis.**

## Parallel vs. Series Capacitance

I have provided you with multi-meters capable of measuring capacitance and I have two capacitors that you can use here. First, measure each of the capacitance and try to get some rough agreement between the meter reading and what is printed on the side of the capacitor. After you have done this, use the clip provided to connect one leg of the capacitance in series. Measure this capacitance. Then, place both of the capacitance into the slot of the meter and then measure the parallel capacitance. Determine the percentage error in each of the measurements. You will probably find that the error is higher than it was for the resistance. Part of the reason for this is that it is simply harder to measure capacitance than resistance.

You may have noticed that the capacitor has markings on its side. There is not really only one standard for capacitance rating through markings. If you are diligent enough, you will find that there are some systems in wide use today but my absolute best advice to you is if you are going to use a capacitor, either measure the capacitance or get one that has the values printed on the side. While I am at it, I need to mention something about the polarity of capacitors. Modern capacitors require you to respect polarity, keeping + sides with the higher potentials. If you fail to connect your capacitor in many circuits properly, you will sooner or later get a big surprise. The surprise will be the failure of the capacitor. This is accompanied by an enormous pop and a release of energy. More annoying, however, is that it will scare you without warning. The moral to this story is to get your capacitor polarities correct.

**You are required to perform a calculation of the equivalent capacitance for 2 dissimilar capacitance, in series and in parallel (which you measured) today (by hand, with paper) and to show that you can do this calculation. You are required to show it to me so that I can confirm that you have done this. You will do a similar calculation on the sample calculations for today. Be sure to follow through with units.** Compare your calculated result to the measured result (for series and also for parallel). If the results do not agree to within a few percent, check your calculations. Then answer this question: how do capacitors in series add and how do capacitors in parallel add (based upon your measurements).

### Summary of the lab procedure:

measure emf of battery
Measure $V_T$ for R1, R2 , R3, and R
enter the data into internal resistance helper (remember to the emf also) for analysis.

Construct the RC charge circuit.
Measure the RC charge I and V for about 5 minutes. & enter into charge helper for analysis.

Make a circuit diagram of the discharging RC circuit.
Measure the RC discharge I and V for about 5 minutes & enter into discharge helper for analysis.
Observe the emf from the ketchup packet.

Make measurements of C1, C2, (C1 parallel C2) , and (C1 series C2); do the hand calculations and compare to the measured value by %deviation.