Physics 220: UnQuiz 02

A sphere of radius b has a total charge -Q. Inside at the center is a charge +2Q. The sphere otherwise has a uniform charge density. Find the vector electric field for $r<b$ and $r>b$.

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Gauss's law says: $\Phi_E = \frac{Q_{enclosed}}{\epsilon_0}$ Choose a Guassian surface centered on the sphere. We need to calculate the volume charge density.

For the sphere: $\rho = \frac{Q}{\text{volume}} = \frac{-Q}{\frac{4}{3}\pi b^3}$

On the Gaussian Surface, (1) $\vec{E} = |\vec{E}|\hat{r}$, $\vec{E} \parallel \vec{A}$ and (2) $|\vec{E}| = \text{constant}$.

(1) $r \leq b$: $Q_{enclosed} = 2Q + \frac{4}{3}\pi r^3(\rho)$ $\sum_{\Delta A_i} \vec{E}_i \cdot \Delta \vec{A}_i = \sum_{\Delta A_i} E(\Delta A_i) = E \sum_{\Delta A_i} \Delta A_i = E(4\pi r^2)$

$$E_{r\leq b}(4\pi r^2) = \frac{\frac{4}{3}\pi r^3 \rho}{\epsilon_0} + \frac{2Q}{\epsilon_0} \Rightarrow \vec{E}_{r\leq b} = \frac{2Q}{4\pi\epsilon_0 r^2} + \frac{r\rho}{\epsilon_0} = \frac{Q}{2\pi\epsilon_0 r^2} - \frac{rQ}{\frac{4}{3}\pi b^3 \epsilon_0} = \frac{Q}{2\pi\epsilon_0 r^2} - \frac{3rQ}{4\pi\epsilon_0 b^3}$$

$$\vec{E}_{r\leq b} = \left[\frac{Q}{2\pi\epsilon_0 r^2} - \frac{3rQ}{4\pi\epsilon_0 b^3}\right]\hat{r}$$

(2) $r \geq b$: In the case all the charge is enclosed. Thus the total charge enclosed is given by : -Q + 2Q = Q.
On the Gaussian Surface, (1) $\vec{E} = |\vec{E}|\hat{r}$, $\vec{E} \parallel \vec{A}$ and (2) $|\vec{E}| = \text{constant}$.

$Q_{enclosed} = Q$ $\sum_{\Delta A_i} \vec{E}_i \cdot \Delta \vec{A}_i = \sum_{\Delta A_i} E(\Delta A_i) = E \sum_{\Delta A_i} \Delta A_i = E(4\pi r^2)$

$$E_{r\geq b}(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow E_{r\geq b} = \frac{Q}{4\pi\epsilon_0 r^2} \Rightarrow \vec{E}_{r\geq b} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$