linear dielectrics

for many materials, provided E is not too strong, the polarization is proportional to E:

$$\vec{P}=\epsilon_0 \chi_E \vec{E}$$

where the susceptability is χ_E .
There is a little problem here though: the electric field is due to free charges , externally applied fields and the bound charges that might be formed or are present. For this reason, it might be best to use D since it comes mostly only from free charges so long as the required symmetry is present.

$$\vec{D}=\epsilon_0 \vec{E}+\vec{P}=\epsilon_0 \vec{E}+\epsilon_0 \chi_E \vec{E}=\epsilon_0 \vec{E}\left(1+\chi_E\right)=\epsilon \vec{E};\epsilon\equiv\epsilon_0\left(1+\chi_E\right);\epsilon_r\equiv\frac{\epsilon}{\epsilon_0}=1+\chi_E$$

Example 4.5: Metal sphere of radius a, charge Q, surrounded by linear dielectric to radius b. Find the potential at the center.

$$\oiint \vec{D}\cdot d\vec{A}=Q_f\Rightarrow\vec{D}=\frac{Q}{4\pi r^2}\hat{r}\Rightarrow r>b:\vec{E}=\frac{Q}{4\pi\epsilon_0 r^2}\hat{r};a<r<b:\vec{E}=\frac{Q}{4\pi\epsilon r^2}\hat{r};r<a:D=E=P=0$$

$$V=-\int_{\infty}^{b}\frac{Q}{4\pi\epsilon_0 r^2}dr-\int_{b}^{a}\frac{Q}{4\pi\epsilon r^2}dr=\frac{Q}{4\pi}\left[\frac{1}{\epsilon_0 b}-\frac{1}{\epsilon a}+\frac{1}{\epsilon b}\right]$$

Inside;

$$\vec{P}=\frac{\left(\epsilon-\epsilon_0\right)_E Q}{4\pi\epsilon_0 r^2}\hat{r}$$