Waves Good (Physics and way too condensed)

wave function:

$$f(z,t) = A\cos[k(z - vt) + \delta]$$

A = amplitude, k = wave number, v=wave speed, δ is the phase constant.

wavelength: $\lambda = \frac{2\pi}{k}$ period: $T = \frac{2\pi}{kv}$ frequency : $f = \frac{1}{T} = \frac{kv}{2\pi} = \frac{v}{\lambda}$

Note: I avoid the use of the Greek letter ν to represent frequency for at least two reasons: (1) it looks a whole lot like v and (2) when you are just learning physics and are not used to it, the letter f is much more reasonable. I suggest you mark though all letters that are in a text with f like this when you see them.

$$\omega = 2\pi f = \frac{v}{\lambda} : v = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{\omega}{k}$$

Complex Notation

We want to represent $f(z,t) = A\cos(kz - \omega t + \delta)$ using complex notation.

This can be done by:

$$f(z,t) = A\,\Re\left[e^{i(kz - \omega t + \delta)}\right]$$

Where

$$e^{i\theta} = \cos\theta + i\sin\theta \,;\, i \equiv \sqrt{-1}$$

The complex wave function is given by:

$$\tilde{f}(z,t) = \tilde{A}e^{i(kz - \omega t)}$$

Where the phase constant is absorbed by :

$$\tilde{A} \equiv Ae^{i\delta}$$

and the actual wave function is given by:

$$f(z,t) = \Re\left[\tilde{f}(z,t)\right]$$

Linear superposition of waves:

$$\tilde{f}(k,z) = \int_{-\infty}^{+\infty} \tilde{A}(k)e^{i(kz - \omega t)}dk$$

which allows waves to run in both directions.

Boundary Conditions

Suppose in a string, a wave from - infinity is incident on a boundary at zero:

$$\tilde{f}_I = \tilde{A}_I e^{i(k_1 z - \omega t)}$$

Part of the wave will be reflected:

$$\tilde{f}_R = \tilde{A}_R e^{i(-k_1 z - \omega t)}$$

Part of the wave will be transmitted:

$$\tilde{f}_T = \tilde{A}_T e^{i(k_2 z - \omega t)}$$

Over all space, the wave functions are then given by:

$$\tilde{f}(z,t) = \begin{bmatrix} \tilde{A}_I e^{i(k_1 z - \omega t)} + \tilde{A}_R e^{i(-k_1 z - \omega t)} & z<0 \\ \tilde{A}_T e^{i(k_2 z - \omega t)} & z>0 \end{bmatrix}$$

Without pursuing the excellent arguments of your author, you can easily see (page 371)

$$f(0_-,t) = f(0_+,t) \quad \text{and} \quad \left[\frac{\partial f}{\partial z}\right]_{0_-} = \left[\frac{\partial f}{\partial z}\right]_{0_+} \quad .$$

This allows solution for the (complex) amplitudes:

$$\tilde{A}_I + \tilde{A}_R = \tilde{A}_T \;:\; k_1(\tilde{A}_I - \tilde{A}_R) = k_2 \tilde{A}_T$$

Solutions: in terms of k:

$$\tilde{A}_R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)\tilde{A}_I \quad : \quad \tilde{A}_T = \left(\frac{2k_1}{k_1 + k_2}\right)\tilde{A}_I$$

In terms of v:

$$\tilde{A}_R = \left(\frac{v_2 - v_1}{v_1 + v_2}\right)\tilde{A}_I \quad : \quad \tilde{A}_T = \left(\frac{2v_2}{v_1 + v_2}\right)\tilde{A}_I$$

With real amplitudes:

$$A_R e^{i\delta_R} = \left(\frac{v_2 - v_1}{v_1 + v_2}\right) A_I e^{i\delta_I} \quad : \quad A_T e^{i\delta_T} = \left(\frac{2v_2}{v_1 + v_2}\right) A_I e^{i\delta_I}$$

If we are specifically talking about strings, $v = \sqrt{\frac{T}{\mu}}$, then

(a) second string lighter than the first so that $v_2 > v_1$ then $\delta_R = \delta_I = \delta_T$: 0 degrees shift.

(b) second string heavier than the first so that $v_2 < v_1$ then $\delta_R + \pi = \delta_I = \delta_T$: 180 degree shift.

(c) second string infinitely massive then $\delta_R + \pi = \delta_T$: rigid, 180 degree phase shift also.