7.3.5 Maxwell's equations in matter

We are going to separate things into contributions from free and bound charges and currents. If you will recall:

$$\rho_b = -\vec{\nabla}\cdot\vec{P} \quad \text{and} \quad \vec{J}_b = \vec{\nabla} \times \vec{B}$$

There is now one additional current which is to be considered: it arises from a system which undergoes a time rate of change of polarization. I can show it from above:

$$\frac{\partial \rho_b}{\partial t} = -\vec{\nabla}\cdot\frac{\partial \vec{P}}{\partial t}$$

From the continuity equation, a change in charge density implies a current:

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho_b}{\partial t} \Rightarrow \vec{\nabla}\cdot\vec{J}_p = \vec{\nabla}\cdot\frac{\partial \vec{P}}{\partial t} \Rightarrow \vec{J}_P = \frac{\partial \vec{P}}{\partial t}$$

So a change in polarization implies there must be a corresponding current density associated with this change. Your author argues this a different way and then shows that the continuity equation is satisfied. I prefer this method. Notice the subscript p on this current to delineate that it came from polarization and is not related to bound currents.

So far, here is what we have: charge is divided into two parts:

$$\rho = \rho_f + \rho_b = \rho_f - \vec{\nabla}\cdot\vec{P}$$

and now currents are divided into 3 parts:

$$\vec{J} = \vec{J}_f + \vec{J}_b + \vec{J}_p = \vec{J}_f + \vec{\nabla} \times \vec{M} + \frac{\partial \vec{P}}{\partial t}$$

Let's look at Gauss's law:

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_f}{\epsilon_0} - \frac{\left(\vec{\nabla}\cdot\vec{P}\right)}{\epsilon_0}$$

But since $\vec{\nabla}\cdot\vec{D}=\rho_f$, we can write: $\vec{\nabla}\cdot\vec{E}=\frac{1}{\epsilon_0}\left[\vec{\nabla}\cdot\vec{D}-\vec{\nabla}\cdot\vec{P}\right]\Rightarrow\vec{D}=\epsilon_0\vec{E}+\vec{P}$.

Let's look at Ampere's law:

$$\vec{\nabla}\times\vec{B}=\mu_0\left(\vec{J}_f+\vec{\nabla}\times\vec{M}+\frac{\partial\vec{P}}{\partial t}\right)+\epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$$

Since $\vec{D}=\epsilon_0\vec{E}+\vec{P}$, we can write this as:

$$\vec{\nabla}\times\vec{B}=\mu_0\left(\vec{J}_f+\vec{\nabla}\times\vec{M}\right)+\mu_0\frac{\partial\vec{D}}{\partial t}$$

Now since

$$\vec{H}=\frac{1}{\mu_0}\vec{B}-\vec{M}\Rightarrow\vec{\nabla}\times\vec{H}=\frac{1}{\mu_0}\vec{\nabla}\times\vec{B}-\vec{\nabla}\times\vec{M}\Rightarrow\vec{\nabla}\times\vec{B}=\mu_0\vec{\nabla}\times\vec{H}+\mu_0\vec{\nabla}\times\vec{M}$$

we can write:

$$\mu_0\vec{\nabla}\times\vec{H}+\mu_0\vec{\nabla}\times\vec{M}=\mu_0\vec{J}_f+\mu_0\vec{\nabla}\times\vec{M}+\mu_0\frac{\partial\vec{D}}{\partial t}\Rightarrow\vec{\nabla}\times\vec{H}=\vec{J}_f+\frac{\partial\vec{D}}{\partial t}$$

So we now have Maxwell's equations for matter:

$$\vec{\nabla}\cdot\vec{D}=\rho_f : \vec{\nabla}\cdot\vec{B}=0 : \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} : \vec{\nabla}\times\vec{H}=\vec{J}_f+\frac{\partial\vec{D}}{\partial t}$$

You can see that D was called the electric displacement and it gives rise to the displacement current. Thus the similarity in names.
You also need the constitutive relations which for linear materials are:

$$\vec{P}=\epsilon_0\chi_E\vec{E} : \vec{M}=\chi_M\vec{H} : \vec{D}=\epsilon\vec{E} : \vec{B}=\mu\vec{H} : \epsilon\equiv\epsilon_0\left(1+\chi_E\right) : \mu=\mu_0\left(1+\chi_M\right) : \vec{J}_D=\frac{\partial\vec{D}}{\partial t}$$