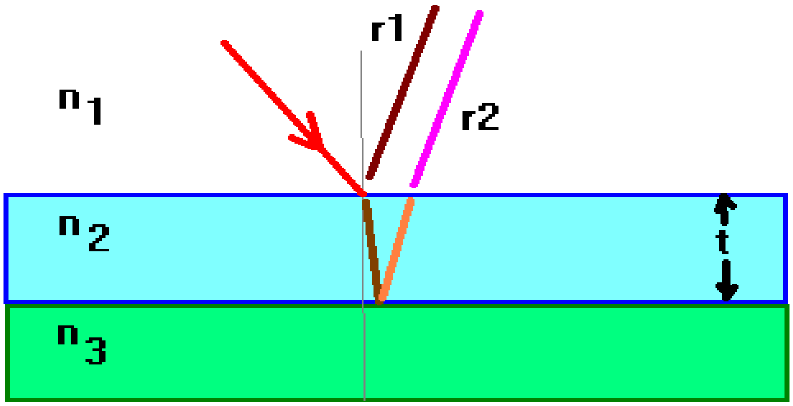
Geometrical Optics Notes
Thin Film interference

It is important to note in this discussion that we are describing NORMAL incidence but the picture shows non-normal incidence for clarity (only).



There are 2 important points to remember when discussing normal thin film interference.

Rule 1: A path difference δ in a medium of index of refraction n is $\boldsymbol{\delta=2nt}$ (t=thickness).

Rule 2: A ray goes through a 0^0 phase shift when going from medium 1 and reflecting off of medium 2 if $n_1>n_2$; otherwise there will be a 180^0 phase shift upon reflection (excluding the case n1=n2 in which case there is no reflection).

Case 1: $n_1<n_2$ and $n_2>n_3$ (this is, for example, a soap film in the air)

Ray r1 goes through a 180^0 phase shift while ray r2 goes through a 2nt path difference.

$$\text{Constructive}: 2n_2t=(m+\tfrac{1}{2})\lambda \text{ for } m=\{0,1,2,3,\ldots\}$$

$$\text{Destructive}: 2n_2t=m\lambda \text{ for } m=\{1,2,3,\ldots\}$$

Case 2: $n_1<n_2$ and $n_2<n_3$ (this is, for example, oil on water)

Ray r1 goes through a 180^0 phase shift while ray r2 goes through a 2nt path difference and a 180^0 phase shift.

$$\text{Constructive}: 2n_2t=m\lambda \text{ for } m=\{1,2,3,\ldots\}$$

$$\text{Destructive}: 2n_2t=(m+\tfrac{1}{2})\lambda \text{ for } m=\{0,1,2,3,\ldots\}$$

The following 2 cases are slightly more difficult because if n1 is not 1, the required path difference in the second medium is relative to the wavelength in the first medium, not air. Let's consider interference which ultimately occurs in medium 1. If in air, light has a wavelength λ, we need to find the wave length in medium 1. At the interface between air and medium 1, the frequency will stay the same, but the velocity changes by:

$$n_1 = \frac{c}{v_1} \Rightarrow v_1 = \frac{c}{n_1}$$

The wavelength in medium 1 is then given by:

$$f\lambda_1 = v_1 \Rightarrow f\lambda_1 = \frac{c}{n_1} \Rightarrow \lambda_1 = \frac{1}{n_1}\frac{c}{f} = \frac{1}{n_1}\lambda_{air}$$

Case 3: $n_1>n_2$ and $n_2>n_3$: Conditions are the same as for Case 2 except the wavelength becomes λ_1.

Ray r1 goes through a 0^0 phase shift while ray r2 goes through a $2n_2t$ path difference.

Case 4: $n_1>n_2$ and $n_2<n_3$: Conditions are the same as for Case 1 except the wavelength becomes λ_1 .

Ray r1 goes through a 0^0 phase shift while ray r2 goes through a $2n_2t$ path difference and a 180^0 phase shift.

Young's Double slit experiment 1801

(experimental proof of the wave nature of light)

The easiest way to understand the resulting interference pattern that results from shining coherent monochromatic light on a double slit is by looking at the path difference between light leaving the two slits. When the path difference is an integer number of wavelengths, a bright band occurs (constructive interference). For a half wavelength path difference, destructive interference occurs (and a dark band results). The bright band positions are given by

$$\tan(\theta) = \frac{\delta}{d} = \frac{Y}{L}$$

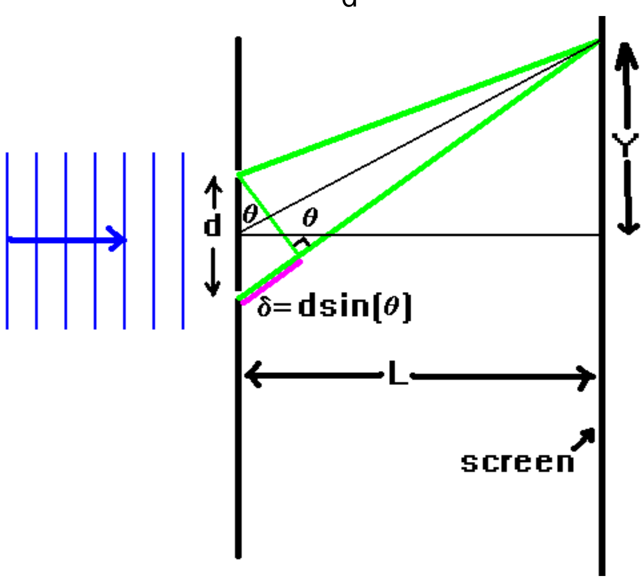
δ is the "path difference" and this is the determining factor in whether the interference is constructive or destructive. The conditions are:

$$\text{Constructive}: \delta = m\lambda \ \{m = 0, \pm 1, \pm 2 \ldots\}$$

$$\text{band positions: } Y = \frac{m\lambda L}{d}$$

$$\text{Destructive}: \delta = (m + \tfrac{1}{2})\lambda \ \{m = 0, \pm 1, \pm 2 \ldots\}$$

$$\text{dark positions: } Y = \frac{(m+\frac{1}{2})\lambda L}{d}$$



I shall not prove this but the relative intensity will be given by

$$\frac{I}{I_0} = \cos^2\left(\frac{\pi d}{\lambda L} Y\right)$$

Remember: Intensity $\equiv \frac{\langle \text{Power} \rangle}{\text{Area}}$