2016	Physics 220: Worksheet 22	Name: ________________

(1) A plane mirror has an object in front of it at a distance of 10 cm. Characterize the image. Then, what is the shortest mirror that will allow you to see your entire height?

(2) A convex mirror has a radius of curvature of 10 cm. An object is placed at 20 cm from the mirror. Characterize the image.

(3) A concave mirror has a radius of curvature of 10 cm. An object is placed at 20 cm from the mirror. Characterize the image.

(4) A concave mirror has a radius of curvature of 10 cm. An object is placed at 5 cm from the mirror. Characterize the image. Then if the object is placed at 2 cm from the mirror, characterize the image. Suppose the object is 1.5 cm high ... how high is the image?

(5) Light shines on a glass slab at an angle of 65^0. If it is observed that the reflected light is completely polarized, then what is the index of refraction of the glass? What is the smallest that Brewster's angle can be?

(1) A plane mirror has an object in front of it at a distance of 10 cm. Characterize the image. Then, what is the shortest mirror that will allow you to see your entire height?

Solution: A plane mirror can be made by allowing an infinite radius of curvature. This then gives us a focal length of zero. If this is true, then the mirror equation shows $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}=0 \Rightarrow s'=-s$. The magnification is given by $M=-\frac{s'}{s}=-\frac{-1}{1}=+1$. The image is [virtual: upright : unmagnified]. For the answer to part two, consider the "alien planet" discussion in class.

(2) A convex mirror has a radius of curvature of 10 cm. An object is placed 20 cm from the mirror. Characterize the image.

Solution: Since the mirror is convex, one must stand behind the mirror in order to sketch the surface. This means that R<0 and thus f=-5 cm. The mirror equation becomes:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{s'}=-\frac{1}{5}-\frac{1}{20}=-\left(\frac{4+1}{20}\right)=-\frac{5}{20}=-\frac{1}{4}$$

So s'=-4. The magnification is M=-(-4/20)=+1/5. The image is characterized by [virtual : upright : reduced]. This characterization is true for all convex mirrors and is the only case that can occur for single element systems.

Why do car manufactures can print that statement on the newer passenger side mirrors? Let me discuss this statement.

The first thing that you'll notice is that the image is reduced (always) in a concave mirror. In fact, I can provide you with a general result, so let me do so. The image position is always given by: $\frac{1}{s'}=-\left[\frac{1}{|f|}+\frac{1}{|s|}\right]=-\left[\frac{|s|+|f|}{|s||f|}\right] \Rightarrow s'=-\frac{|s||f|}{|s|+|f|}$ and thus the magnification will always be given by $M=-\frac{s'}{s}=\frac{|f|}{|s|+|f|}$. Thus technically said, the image of the object is 100% of the time at less of a distance from the mirror than the object is. The resolution to the problem is thus not found here. Instead, we need to look at how you interpret the image that you see. This image is also 100% of the time smaller than the object. Without having the side of your car reflecting in the mirror, you would be inclined to think that the image that you see, since it is reduced significantly over what it would be for a plane mirror, is more distant than it actually is. The magnification is always given by: $M=-\frac{s'}{s}=\frac{|f|}{|s|+|f|}$. This is one of the reasons I like to adjust my side mirror so that I can slightly see the side of my car in order to provide a calibration gauge for the distance.

Ok, I know you're going to want an explanation for how the size of an object varies in the plane mirror. Here it is:

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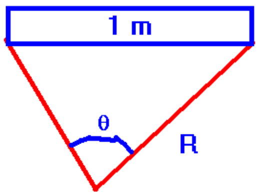
Suppose you look at a meter stick from some distance R. You observe that the meter stick subtends an angle Θ. The length of the meter stick here is 1 meter so that S=Rθ. As the meter stick gets closer, R decreases and thus θ increases. Suppose in your mind, you know when you look in your (planar) rear view mirror that a standard width car appears to be 4 cm in length when it is 1 car length away from you. If you mistakenly thought that the passenger side mirror was a plane mirror, you would see an image which is smaller than the 4 cm image you've become accustomed to for safety … you might measure only a 3 cm image. You would thus mistakenly ignore the fact that the car behind you is closer than is actually within that "safe" single car length. Astronomers often make use of this "parallax" to measure astronomical distances. They make slightly different measurements of the angles, but the idea is the same.



(3) A concave mirror has a radius of curvature of 10 cm. An object is placed at 20 cm from the mirror. Characterize the image.

Solution: Since the mirror is concave, one must stand in front of the mirror in order to sketch the surface. This means that R>0 and thus f=+5 cm. The mirror equation becomes:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=+\frac{1}{5}-\frac{1}{20}=\left(\frac{+4-1}{20}\right)=+\frac{3}{20}=0.15$$

So s'=+20/3=+6.667. The magnification is M=-(20/3/20)=-.3333. This image is characterized by [real : inverted : reduced].

(4) A concave mirror has a radius of curvature of 10 cm. An object is placed at 5 cm from the mirror. Characterize the image. Then if the object is placed at 2 cm from the mirror, characterize the image. Suppose the object is 1.5 cm high … how high is the image?

Solution: Since the mirror is concave, one must stand in front of the mirror in order to sketch the surface. This means that R>0 and thus f=+5 cm. The mirror equation becomes:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=+\frac{1}{5}-\frac{1}{5}=0\Rightarrow s'=\infty$$

If the object is placed at the focal point of a concave mirror, no image forms (and we now know that it is really not correct to say an image forms at infinity, because light does not get there fast enough). On the other hand, if an object is placed at infinity, the image will form at the focal point of the mirror.

Now if the object is placed at 2 cm from the mirror,

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=+\frac{1}{5}-\frac{1}{2}=\frac{2}{10}-\frac{5}{10}=\frac{-3}{10}\Rightarrow s'=-10/3$$

The magnification is M=-(-10/3/2)=+1.6. The resulting image is [virtual : upright: enlarged]. If the object was initially 1.5 cm high, the image would be 2.4 cm high.

(5) Light shines on a glass slab at an angle of 65^0. If it is observed that the reflected light is completely polarized, then what is the index of refraction of the glass? What is the smallest that Brewster's angle can be?

This problem involves working with Brewster's angle. At this particular angle of incidence, two very significant things happen: (a) the reflected light is completely polarized and (b) the angle between the transmitted ray and the reflected ray is exactly 90^0. This angle is often called θ_B (or θ_p). You will experiment with this in lab.

Solution: Since the reflected light is completely polarized, the angle of incidence must be the Brewster angle. Therefore the angle between the reflected ray and the transmitted ray must be 90^0. We have from Snell's law and the law of reflection that at θ_B (or θ_p), $\sin(\theta_B)=\sin(\theta_r)=n\sin(\theta_t)$. Now, since the angle between θ_r and θ_t is 90^0, we have that $\theta_t=90^0-\theta_B$ so that $\sin(\theta_t)=\sin(90^0-\theta_B)=\cos(\theta_B)$. We use this in Snell's law to rewrite it as: $\sin(\theta_B)=n\cos(\theta_B)$ which then gives us the result $\tan(\theta_B)=n$. We can thus find the index of refraction easily by taking the tangent of 65^0 or $n=2.14$. Brewster's angle can be no smaller than $\tan(\theta_B)=1$ or 45^0 since this would give an $n<1$.