Physics 220: Unquiz #2

A sphere of uniform charge density ρ and total charge Q has a radius a. At the center of the sphere is a solid metal sphere of radius b, b<a. Find the electric field vector in the three regions.

For r>a, choose a Gaussian sphere of radius r, r>a. On the Gaussian sphere, the electric field is parallel to the normal to the area and uniform. The total charge enclosed is Q. By Gauss's law:

$$\Phi_E=\frac{Q_{enc}}{\epsilon_0}:\Phi_E=\sum_{\Delta A_i}\vec{E}_i\cdot\Delta\vec{A}_i=\sum_{\Delta A_i}E_i\cdot\Delta A_i=E\sum_{\Delta A_i}\Delta A_i=E\left(4\pi r^2\right):Q_{enc}=Q\Rightarrow E\left(4\pi r^2\right)=\frac{Q}{\epsilon_0}$$

$$\Rightarrow\vec{E}=\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

For r>b, r<a, choose a Gaussian sphere of radius r, r>b, r<a. On the Gaussian sphere, the electric field is parallel to the normal to the area and uniform.

At a radius r,

$$Q_{enc}=\rho\left(\frac{4}{3}\pi r^3-\frac{4}{3}\pi b^3\right)=\frac{4}{3}\pi\rho\left(r^3-b^3\right):\Phi_E=E\left(4\pi r^2\right)\Rightarrow E=\frac{\rho}{3\epsilon_0}\left(r-\frac{b}{r^2}\right)$$

$$\vec{E}_{r>b\ ,\ r<a}=\frac{\rho}{3\epsilon_0}\left(r-\frac{b}{r^2}\right)\hat{r}$$

For r<b, no net charge is enclosed. In general the area here is not zero so the electric field for r<b=0. A solid metal sphere can not have a net electric field in its interior.