Simplest solutions for Laplace's Equation
These solutions really comprise the cases where only one (distance) variable of the problem is involved.

(Cartesian):
Suppose V depends only upon x. Then:

$$\nabla^2 V = 0 \Rightarrow V = a + bx$$

Then fit the boundary conditions:

$$@\, x_1 : V = V_1 \Rightarrow V_1 = a + bx_1$$

$$@\, x_2 : V = V_2 \Rightarrow V_2 = a + bx_2$$

These are easily solved to give:

$$a = -\frac{V_1 x_2 - V_2 x_1}{x_1 - x_2} : b = -\frac{V_2 - V_1}{x_1 - x_2}$$

For example: the parallel plate capacitor, at x=0, V=0 and at x=d, V=V:

$$a = 0 : b = \frac{V}{d} \Rightarrow V(x) = 0 + \frac{V}{d} x$$

(cylindrical)
Suppose V depends only upon s. Then:

$$\nabla^2 V = 0 \Rightarrow \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) = 0 \Rightarrow \frac{d}{ds}\left(s\frac{dV}{ds}\right) = 0 \Rightarrow s\frac{dV}{ds} = a_1 \Rightarrow \frac{dV}{ds} = \frac{a_1}{s} \Rightarrow V(s) - V(s_0) = a_1 \ln\left[\frac{s}{s_0}\right]$$

$$\Rightarrow V(s) = V(s_0) + \ln\left[\frac{s}{s_0}\right]^{a_1}$$

$$@\, s_0 : V = V_0 \Rightarrow \text{No new info}$$

$$@\, s_1 : V = V_1 \Rightarrow V_1 = V_0 + a_1 \ln\left[\frac{s_1}{s_0}\right] \Rightarrow a_1 = \frac{V_1 - V_0}{\ln\left[\frac{s_1}{s_0}\right]} \Rightarrow V = V_0 + \left[\frac{V_1 - V_0}{\ln\left[\frac{s_1}{s_0}\right]}\right]\ln\left[\frac{s}{s_0}\right]$$

For example: the coaxial cable: at s=a, V=0 and at s=b, V=V

$$V = \left[\frac{V}{\ln\left[\frac{b}{a}\right]}\right]\ln\left[\frac{s}{a}\right]$$

(spherical)
Suppose V depends only upon r. Then:

$$\vec{\nabla}^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)$$

$$\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = 0 \Rightarrow r^2\frac{dV}{dr} = a \Rightarrow \frac{dV}{dr} = \frac{a}{r^2} \Rightarrow V(r) = V(r_1) - a\left[\frac{1}{r} - \frac{1}{r_1}\right] \Rightarrow a = -\frac{V_b - V_a}{\left[\frac{1}{b} - \frac{1}{a}\right]}$$

$$\Rightarrow V(r) = V_a + \frac{V_b - V_a}{\left[\frac{1}{b} - \frac{1}{a}\right]}\left[\frac{1}{r} - \frac{1}{r_1}\right]$$

For example: the concentric spheres: at r=a, V=V_1 and at r=b, V=-V_1:

$$V(r) = V_1 - \frac{2V_1}{\left[\frac{1}{b} - \frac{1}{a}\right]}\left[\frac{1}{r} - \frac{1}{a}\right].$$