Let's try a model for the Hydrogen Atom

Assume you have a charge distribution given by:
a point charge q at r=0 and

$$\rho_0 e^{-\frac{r^2}{a^2}}$$

for r>a.

The overall charge is zero, so you'll need to fix this. We require:

$$+q+4\pi\int_{r=a}^{\infty}\rho_0 e^{-\frac{r^2}{a^2}}\left(r^2 dr\right)=0$$

We require:

$$\rho_0=-\frac{q}{4\pi\int_{r=a}^{\infty}e^{-\frac{r^2}{a^2}}r^2 dr}\Rightarrow\rho_0=-\frac{q}{4\pi a^3\int_{\eta=1}^{\infty}e^{-\eta^2}\eta^2 d\eta}$$

This is an integral which can be done in terms of the error function.
At our favorite site, enter:
x^2 Exp[-x^2]
The result is:

$$-\tfrac{1}{2}xe^{-x^2}+\tfrac{1}{4}\sqrt{\pi}\mathrm{Erf}[x]$$

For our integral, the result is:

$$-\tfrac{1}{2}\eta e^{-\eta^2}+\tfrac{1}{4}\sqrt{\pi}\mathrm{Erf}[\eta]\Big|_1^{\infty}=\tfrac{\sqrt{\pi}}{4}+\tfrac{e^{-1}}{2}-\tfrac{\sqrt{\pi}}{4}\mathrm{Erf}\left[1\right]$$

The error function of (1) is on the order of ½. Look at:
http://mathworld.wolfram.com/Erf.html
Now we can find the potential from this:

$$V\left(\vec{r}_p\right)=\int_{r=a}^{r=\infty}\int_{\theta=0}^{\theta=\pi}k\rho_0\frac{e^{-\frac{r^2}{a^2}}}{\left|\vec{r}-z_p\hat{z}\right|}r^2 dr+\frac{kq}{r_p}$$

I think it's pretty clear now what to do … we had:

$$V\left(\vec{r}_p\right)=2\pi\int_{r=a}^{r=\infty}\int_{\theta=0}^{\theta=\pi}k\rho_0\frac{e^{-\frac{r^2}{a^2}}}{\sqrt{x^2+y^2+\left(z_p-z\right)^2}}r^2\sin\left(\theta\right)drd\theta+\frac{kq}{r_p}$$

$$V_1\left(\vec{r}_p\right)=2\pi\int_{r=a}^{r=\infty}\int_{\theta=0}^{\theta=\pi}k\rho_0\frac{e^{-\frac{r^2}{a^2}}}{\sqrt{x^2+y^2+\left(z_p-z\right)^2}}r^2\sin\left(\theta\right)drd\theta$$

$$V_1\left(\vec{r}_p\right) = 2\pi \int_{r=a}^{r=\infty} \int_{\theta=0}^{\theta=\pi} k\rho_0 \frac{e^{-\frac{r^2}{a^2}}}{\sqrt{x^2+y^2+z^2-2zz_p+z_p^2}} r^2 \sin(\theta) dr d\theta$$

$$V_1\left(\vec{r}_p\right) = k\rho_0 2\pi \int_{r=a}^{r=\infty} \int_{\theta=0}^{\theta=\pi} \frac{e^{-\frac{r^2}{a^2}}}{\sqrt{r^2-2rz_p \cos(\theta)+z_p^2}} r^2 \sin(\theta) dr d\theta$$

let

$x = 2rz_p \cos(\theta) : dx = -2rz_p \sin(\theta) d\theta : \theta = 0 \Rightarrow x = 2rz_p : \theta = \pi \Rightarrow z = -2rz_p$

$$V_1\left(\vec{r}_p\right) = -\frac{k\rho_0\pi}{z_p} \int_{r=a}^{r=\infty} \int_{x=2rz_p}^{x=-2rz_p} \frac{e^{-\frac{r^2}{a^2}}}{\sqrt{r^2-x+z_p^2}} r dr dx$$

at our favorite site, enter: (b-x)^(-1/2)
then,

$$V_1\left(\vec{r}_p\right) = 2\frac{k\rho_0\pi}{z_p} \int_{r=a}^{r=\infty} e^{-\frac{r^2}{a^2}} \sqrt{r^2 - x + z_p^2}\Bigg|_{x=2rz_p}^{x=-2rz_p} r dr$$

$$V_1\left(\vec{r}_p\right) = 2\frac{k\rho_0\pi}{z_p} \int_{r=a}^{r=\infty} e^{-\frac{r^2}{a^2}} \left[\sqrt{r^2 + 2rz_p + z_p^2} - \sqrt{r^2 - 2rz_p + z_p^2}\right] r dr$$

$$V_1\left(\vec{r}_p\right) = 2\frac{k\rho_0\pi}{z_p} \int_{r=a}^{r=\infty} e^{-\frac{r^2}{a^2}} \left[\left(r + z_p\right) - \begin{matrix} \left(r - z_p\right) \text{ if } r>z_p \\ \left(z_p - r\right) \text{ if } r<z_p \end{matrix}\right] r dr$$

$$V_1\left(\vec{r}_p\right) = 2\frac{k\rho_0\pi}{z_p} \int_{r=a}^{r=\infty} e^{-\frac{r^2}{a^2}} \left[\left(r + z_p\right) - \begin{matrix} \left(r - z_p\right) \text{ if } r>z_p \\ \left(z_p - r\right) \text{ if } r<z_p \end{matrix}\right] r dr$$

This integral has got to be broken into 2 parts then, since in the interval both conditions are true.

Thus:

$$V_1\left(\vec{r}_p\right) = 2\frac{k\rho_0\pi}{z_p}\int_{r=a}^{r=z_p} e^{-\frac{r^2}{a^2}}\left[\left(r+z_p\right)-\left(z_p-r\right)\right]rdr +$$

$$2\frac{k\rho_0\pi}{z_p}\int_{r=z_p}^{r=\infty} e^{-\frac{r^2}{a^2}}\left[\left(r+z_p\right)-\left(r-z_p\right)\right]rdr$$

$$V_1\left(\vec{r}_p\right) = 4\frac{k\rho_0\pi}{z_p}\int_{r=a}^{r=z_p} e^{-\frac{r^2}{a^2}} r^2 dr + 4\frac{k\rho_0\pi}{z_p}\int_{r=z_p}^{r=\infty} e^{-\frac{r^2}{a^2}} z_p r dr$$

These integrals can be simplified somewhat by a scale factor:

$$\eta = \tfrac{r}{a} \Rightarrow a d\eta = dr : r = z_p \Rightarrow \eta = \tfrac{z_p}{a} : r = a \Rightarrow \eta = 1 : r = \infty \Rightarrow \eta = \infty$$

$$V_1\left(\vec{r}_p\right) = 4\frac{k\rho_0\pi}{z_p}\int_{\eta=1}^{\eta=\frac{z_p}{a}} e^{-\eta^2} a^3\eta^2 d\eta + 4\frac{k\rho_0\pi}{z_p}\int_{\eta=\frac{z_p}{a}}^{\eta=\infty} e^{-\eta^2} z_p a^2 \eta d\eta$$

$$V_1\left(\vec{r}_p\right) = 4\pi a^3 \frac{k\rho_0}{z_p}\int_{\eta=1}^{\eta=\frac{z_p}{a}} e^{-\eta^2}\eta^2 d\eta + 4\pi a^2 k\rho_0\int_{\eta=\frac{z_p}{a}}^{\eta=\infty} e^{-\eta^2}\eta d\eta$$

The second integral is straight-forward:

enter x (E^(-x^2)) at our favorite site

$$\int_{\eta=\frac{z_p}{a}}^{\eta=\infty} e^{-\eta^2}\eta d\eta = -\frac{e^{-\eta^2}}{2}\Bigg|_{\eta=\frac{z_p}{a}}^{\eta=\infty} = -\frac{1}{2}\left[0 - e^{-\frac{z_p^2}{a^2}}\right] = \frac{1}{2}e^{-\frac{z_p^2}{a^2}}$$

The first integral is also not all that bad:

$$\int_{\eta=1}^{\eta=\frac{z_p}{a}} e^{-\frac{r^2}{a^2}}\eta^2 d\eta = -\eta\frac{e^{-\eta^2}}{2} + \frac{\sqrt{\pi}}{4}\mathrm{Erf}\left[\eta\right]\Bigg|_{\eta=1}^{\eta=\frac{z_p}{a}}$$

$$\int_{\eta=1}^{\eta=\frac{z_p}{a}} e^{-\frac{r^2}{a^2}}\eta^2 d\eta = -\frac{z_p}{a}\frac{e^{-\frac{z_p^2}{a^2}}}{2} + \frac{\sqrt{\pi}}{4}\text{Erf}\left[\frac{z_p}{a}\right] + \frac{e^{-1}}{2} - \frac{\sqrt{\pi}}{4}\text{Erf}[1]$$

We thus have the potential V_i:

$$V_1 = 4\pi a^2 k\rho_0 \frac{1}{2} e^{-\frac{z_p^2}{a^2}} + 4\pi a^3 \frac{k\rho_0}{z_p}\left[-\frac{z_p}{a}\frac{e^{-\frac{z_p^2}{a^2}}}{2} + \frac{\sqrt{\pi}}{4}\text{Erf}\left[\frac{z_p}{a}\right] + \frac{e^{-1}}{2} - \frac{\sqrt{\pi}}{4}\text{Erf}[1]\right]$$

$$V_1 = 4\pi a^3 \frac{k\rho_0}{z_p}\left[\frac{\sqrt{\pi}}{4}\left(\text{Erf}\left[\frac{z_p}{a}\right] - \text{Erf}[1]\right) + \frac{e^{-1}}{2}\right]$$

Finally, let's add onto this the potential of the positive charge located at the origin:

$$V = \frac{kq}{z_p} + 4\pi a^3 \frac{k\rho_0}{z_p}\left[\frac{\sqrt{\pi}}{4}\left(\text{Erf}\left[\frac{z_p}{a}\right] - \text{Erf}[1]\right) + \frac{e^{-1}}{2}\right]$$

Let's now realize spherical symmetry to obtain:

$$V\left(\vec{r}_p\right) = \frac{kq}{r_p} + 4\pi a^3 \frac{k\rho_0}{r_p}\left[\frac{\sqrt{\pi}}{4}\left(\text{Erf}\left[\frac{r_p}{a}\right] - \text{Erf}[1]\right) + \frac{e^{-1}}{2}\right]$$

This form is valid so long as $r_p > a$. You'll need to do more calculations if you go inside the radius a.

Oh … remember also that

$$\rho_0 = -\frac{q}{4\pi a^3 \int_{\eta=1}^{\infty} e^{-\eta^2}\eta^2 d\eta}$$

If you let $I \approx \int_{\eta=1}^{\infty} e^{-\eta^2}\eta^2 d\eta$

Then

$$\rho_0 = -\frac{q}{4\pi a^3 I_1}$$

and the potential would appear as:

$$V\left(\vec{r}_p\right) = \frac{kq}{r_p}\left[1 - \frac{1}{I_1}\left[\frac{\sqrt{\pi}}{4}\left(\text{Erf}\left[\frac{r_p}{a}\right] - \text{Erf}[1]\right) + \frac{e^{-1}}{2}\right]\right]$$