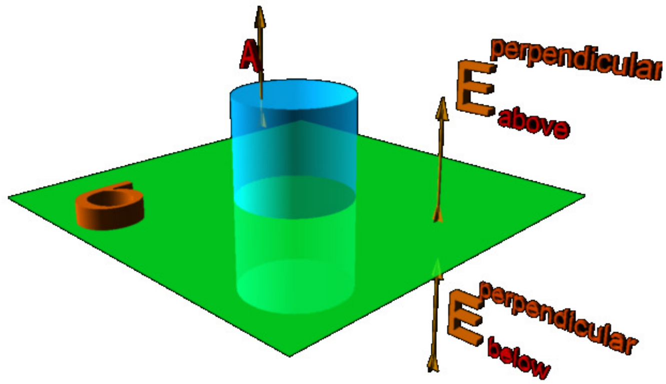
Boundary Conditions when crossing a surface charge σ (2.3.5)
For a surface:



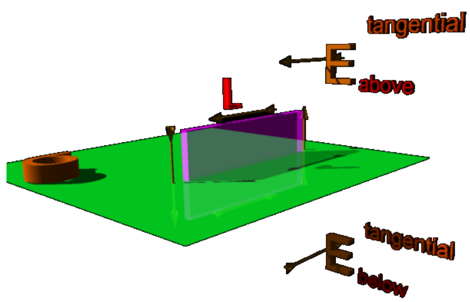
Per Gauss's law:

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \Rightarrow E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\varepsilon_0}$$

(the normal component of E is discontinuous)

Now to get the tangential component, consider Stokes' Theorem:

$$\int_{surface} \left(\vec{\nabla} \times \vec{V}\right) \bullet d\vec{A} = \oint_{path} \vec{V} \bullet d\vec{l}$$



The contribution due to the endspieces vanishes leaving only the top and the bottom.
Since the curl of E vanishes, we only can conclude:

$$E^{\parallel}_{above} = E^{\parallel}_{below}$$

(the tangential component of E is continuous)

We can combine these conditions to read:

$$\vec{E}_{above} - \vec{E}_{below} = \frac{\sigma}{\varepsilon_0} \hat{n}$$

We can show that the potential is continuous across the boundary by considering a path that we let shrink to zero:

$$\vec{E}_{above} \bullet \delta\vec{L}_{above} - \vec{E}_{below} \bullet \delta\vec{L}_{below} = \frac{\sigma}{\varepsilon_0} \delta L$$

$$\Delta V = \lim_{\delta L \to 0} \frac{\sigma}{\varepsilon_0} \delta L = 0$$

(the potential is continuous across the surface charge density)

The gradient in V however does reflect the discontinuity:

$$\vec{\nabla}\left[\Delta V\right] = \vec{\nabla} V_{above} - \vec{\nabla} V_{below} = -\frac{\sigma}{\varepsilon_0} \hat{n}$$

This is also written as:

$$\frac{\partial V_{above}}{\partial n} - \frac{\partial V_{below}}{\partial n} = -\frac{\sigma}{\varepsilon_0} : \frac{\partial V}{\partial n} \equiv \vec{\nabla} V \bullet \hat{n}$$

(this is called the normal derivative)

You can write the tangential derivative as:

$$\vec{\nabla} V \times \hat{n}$$