1.49

(a)

$\vec{F}_1 = x^2\hat{z} : \vec{F}_2 = x\hat{x} + y\hat{y} + z\hat{z}$

$\vec{\nabla} \bullet \vec{F}_1 = 0 : \vec{\nabla} \bullet \vec{F}_2 = 3$

$$\vec{\nabla} \times \vec{F}_1 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & x^2 \end{vmatrix} = \hat{x}(0) - \hat{y}(2x) + \hat{z}(0) = -2x\hat{y}$$

$$\vec{\nabla} \times \vec{F}_2 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = \hat{x}(0) - \hat{y}(0) + \hat{z}(0) = \vec{0}$$

Consider: $f = \frac{1}{2}\left[x^2 + y^2 + z^2\right] \Rightarrow \vec{\nabla} f = \vec{F}_2$

A scalar potential is then: $V = -f$

$$\text{Consider: } \vec{g} = \tfrac{1}{3}x^2\hat{y} \Rightarrow \vec{\nabla} \times \vec{g} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \frac{1}{3}x^3 & 0 \end{vmatrix} = 0\hat{x} - 0\hat{y} + x^2\hat{z} = \vec{F}_1$$

A vector potential is then $\vec{A} = \vec{g}$

(b)

Consider: $f = xyz \Rightarrow \vec{\nabla} f = yz\hat{x} + xz\hat{y} + xy\hat{z} \Rightarrow V = -[xyz]$

$x : \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = yz \Rightarrow A_z = \frac{1}{2}zy^2 : A_y = f_y(x, y)$

$y : -\frac{\partial A_z}{\partial x} + \frac{\partial A_x}{\partial z} = xz \Rightarrow A_x = \frac{1}{2}xz^2 : A_z = f_z(y, z)$

$z : \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = xy \Rightarrow A_y = \frac{1}{2}yx^2 : A_x = f_x(x, z)$

$\Rightarrow \vec{A} = \frac{1}{2}xz^2\hat{x} + \frac{1}{2}yx^2\hat{y} + \frac{1}{2}zy^2$

Check:

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{1}{2}xz^2 & \frac{1}{2}yx^2 & \frac{1}{2}zy^2 \end{vmatrix} = \hat{x}[zy - 0] - \hat{y}[0 - xz] + \hat{z}[xy] = yz\hat{x} + xz\hat{y} + xy\hat{z}$$

Thus the vector can be written either as the gradient of a scalar or the curl of a vector.

The scalar potential is $V = -[xyz]$

The vector potential is $\vec{A} = \frac{1}{2}xz^2\hat{x} + \frac{1}{2}yx^2\hat{y} + \frac{1}{2}zy^2$