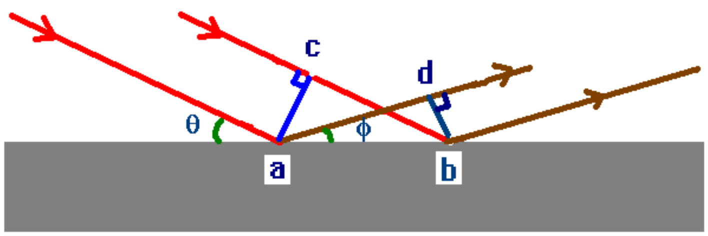
Derivation of the law of reflection from Huygen's principle

Note: Huygen's principle states that at each point of a wavefront, you can treat it as if it is a secondary emitter of light in all directions.



What we need to show is that the two angles indicated above are actually equal.

In a time Δt, the point that was a c travels to b. The reflected light from a travels to d.

This means $\overline{ad} = \overline{cb}$ and, in fact, since the light travels with speed v, we have:

$$\overline{ad} = \overline{cb} = v(\Delta t)$$

The two right triangles (acb and adb) also share the side of length $\overline{ab}$. We can thus find the length of the remaining side:

$$\left(\overline{ac}\right)^2 + \left(\overline{cb}\right)^2 = \left(\overline{ab}\right)^2 \text{ and } \left(\overline{bd}\right)^2 + \left(\overline{ad}\right)^2 = \left(\overline{ab}\right)^2$$

Using the result above you then see clearly:

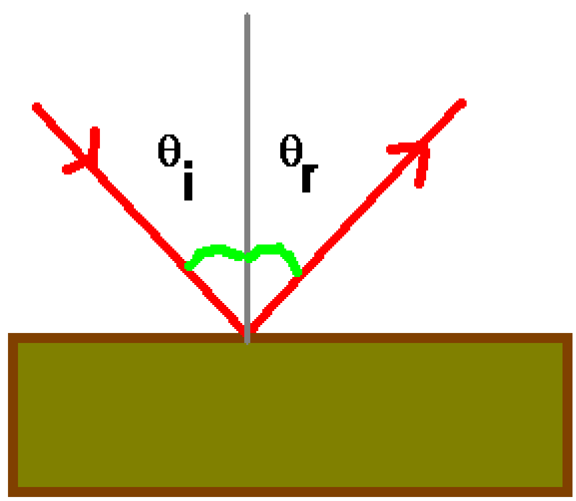
$$\overline{ac} = \overline{bd}$$

Thus the two triangles have sides which are all of exactly the same length.

But, $\overline{ac} = \overline{ab}\sin(\theta) \text{ and } \overline{bd} = \overline{ab}\sin(\varphi) \Rightarrow \sin(\theta) = \sin(\varphi) \Rightarrow \theta = \varphi$

Now in fact, it is the angle of incidence and the angle of reflection which are the same. These are related by subtraction from 90 degrees. Thus we have the law of reflection:

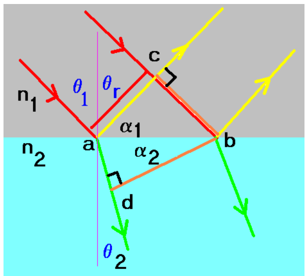
$$\theta_i = \theta_r$$



Derivation of Snell's law

There are other ways to derive the same result without using a reflected wave. I think this is, however, the most straight-forward way to obtain this result.

Consider the red incoming wave, the yellow reflected wave and the green refracted wave as shown. Assume it takes a time Δt for a wavefront to completely impact the interface.



Triangle abd and triangle abc share a side and both of them are right triangles as shown.

$$\overline{ac} = \overline{ab}\cos(\alpha_1) \text{ and } \overline{ad} = \overline{ab}\sin(\alpha_2)$$

we also have these lengths given by:

$$\overline{ad} = v_2(\Delta t) \text{ and } \overline{ac} = v_1(\Delta t)$$

We thus have:

$$v_1(\Delta t) = \overline{ab}\cos(\alpha_1) \text{ and } v_2(\Delta t) = \overline{ab}\sin(\alpha_2)$$

Let's divide the two equations to obtain

$$\frac{v_1\Delta t}{v_2\Delta t} = \frac{v_1}{v_2} = \frac{\overline{ab}\cos(\alpha_1)}{\overline{ab}\sin(\alpha_2)} = \frac{\cos(\alpha_1)}{\sin(\alpha_2)}$$

The index of refraction, however is defined by n=c/v. If we write this in terms of the index of refraction, then we have:

$$\frac{n_2}{n_1} = \frac{\cos(\alpha_1)}{\sin(\alpha_2)}$$

Rearrange this to get:

$$n_1\cos(\alpha_1) = n_2\sin(\alpha_2)$$

The final step is to recognize that

$\cos(\alpha_1)=\sin(\theta_r)=\sin(\theta_1)$ and $\sin(\alpha_2)=\sin(\theta_2)$

Thus:

$$n_1\sin(\theta_1) = n_2\sin(\theta_2)$$

which is Snell's law.

Total internal reflection

In Snell's law let $n_1>n_2$. Then in a slightly different form, Snell's law appears as

$$\sin(\theta_2) = \frac{n_1}{n_2}\sin(\theta_1)$$

The largest that $\sin(\theta_2)$ can be is 1. At this angle let θ_1 be called θ_c (the critical angle). Then

$$\sin(\theta_c) = \frac{n_2}{n_1}$$

For angles greater than this, light will undergo total internal reflection, not refraction. This only happens when light hits an interface from a region of a high index of refraction towards a region of lower index of refraction.