

(1) A lens produces a virtual image of an object which is upright and 2 cm high when a 1 cm high object is placed 20 cm from the lens. What is the focal length of the lens and what is the image position?

(2) A lens produces a real image of an object which is inverted and 2 cm high when a 1 cm high object is placed 20 cm from the lens. What is the focal length of the lens and what is the image position?

(3) Some examples of virtual objects:

Suppose a virtual object is located 10 cm behind a diverging lens with a focal length of 5 cm. Characterize the resulting image.

(4) Suppose a virtual object is located 10 cm behind a converging lens with a focal length of 5 cm. Characterize the resulting image.

(5) Where must an object be located if a real image at 10 cm results from a diverging lens with a focal length of 15 cm? Characterize the resulting image if the object is upright.

(6) An object is at a distance s_0 from a converging lens of focal length f ($s_0 > f$). The object is moving towards the lens with a speed v . How fast is the point at which the image would form be moving at all times?

(1) A lens produces a virtual image of an object which is upright and 2 cm high when a 1 cm high object is placed 20 cm from the lens. What is the focal length of the lens and what is the image position?

$$M = \frac{h'}{h} = +2 = -\frac{s'}{s} = -\frac{s'}{+20} \Rightarrow s' = -40$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{+20} - \frac{1}{40} = \frac{2-1}{40} = \frac{1}{40} = \frac{1}{f} \Rightarrow f = +40 \text{ cm}$$

Notice that it's not possible here to have a real image in this situation.

(2) A lens produces a real image of an object which is inverted and 2 cm high when a 1 cm high object is placed 20 cm from the lens. What is the focal length of the lens and what is the image position?

$$M = \frac{h'}{h} = -2 = -\frac{s'}{s} = -\frac{s'}{+20} \Rightarrow s' = +40$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{+20} + \frac{1}{+40} = \frac{2+1}{40} = \frac{3}{40} = \frac{1}{f} \Rightarrow f = +\frac{40}{3} = +13.3 \text{ cm}$$

(3) Some examples of virtual objects:

Suppose a virtual object is located 10 cm behind a diverging lens with a focal length of 5 cm. Characterize the resulting image.

Since the object is virtual, we have $s = -10 \text{ cm}$. This could have been created as the result of a second lens. Now we can find the image position from the thin lens equation:

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-5} - \frac{1}{-10} = -\frac{2}{10} + \frac{1}{10} = -\frac{1}{10} \Rightarrow s' = -10 \text{ cm}$$

$$M = -\frac{s'}{s} = -\frac{-10}{-10} = -1$$

The image is : [virtual : unmagnified : inverted]

(4) Suppose a virtual object is located 10 cm behind a converging lens with a focal length of 5 cm. Characterize the resulting image.

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{+5} - \frac{1}{-10} = \frac{2}{10} + \frac{1}{10} = +\frac{3}{10} \Rightarrow s' = +\frac{10}{3} = +3.33 \text{ cm}$$

$$M = -\frac{s'}{s} = -\frac{+3.33}{-10} = +0.333$$

The image is : [real : reduced : upright]

(5) Where must an object be located if a real image at 10 cm results from a diverging lens with a focal length of 15 cm? Characterize the resulting image if the object is upright.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} = \frac{1}{f} - \frac{1}{s'} = \frac{1}{-15} - \frac{1}{+10} = \frac{-2-3}{30} = -\frac{5}{30} = -\frac{1}{6} \Rightarrow s = -6 \text{ cm}$$

$$M = -\frac{s'}{s} = -\frac{+10}{-6} = +1.67$$

The image is: [real : enlarged : upright]

(6) An object is at a distance s_0 from a converging lens of focal length f ($s_0 > f$). The object is moving towards the lens with a speed v . How fast is the point at which the image would form be moving at all times?

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s_0 - vt - f}{f(s_0 - vt)} \Rightarrow s' = \frac{f(s_0 - vt)}{s_0 - vt - f}$$

The speed of the image is given by:

$$v' = \frac{ds'}{dt} = \frac{f(-v)}{s_0 - vt - f} - \frac{f(s_0 - vt)}{[s_0 - vt - f]^2} (-v) = -\frac{fv}{[s_0 - vt - f]} \left\{ 1 - \frac{s_0 - vt}{s_0 - vt - f} \right\} = -\frac{fv}{[s_0 - vt - f]} \left\{ \frac{s_0 - vt - f - s_0 + vt}{[s_0 - vt - f]} \right\} = \frac{f^2 v}{[s_0 - vt - f]^2}$$

At one point, this speed will be infinite. There is not really any information contained in this fact so it does not violate relativity. I have, however, been very careful with my words in the statement of this problem. When does the speed become infinite?

This is when

$$s_0 - vt - f = 0 \Rightarrow vt = s_0 - f \Rightarrow t = \frac{s_0 - f}{v}$$

What is the object location at this time?

$$s = s_0 - vt = s_0 - v \left[\frac{s_0 - f}{v} \right] = s_0 - s_0 + f = f.$$

When the object reaches the focal length, the point at which the image forms would move infinitely fast (it would take light a while to catch up with this point).