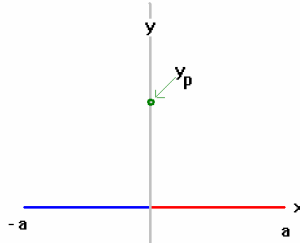


A dipole wire has a total length of  $2a$  and extends from  $x=-a$  to  $x=+a$ . At  $x=0$ , the charge linear distribution changes from  $-\lambda$  to  $+\lambda$ . Find the electric field at points along the symmetric  $y$  axis.



$$\vec{E}_p = \int_q \frac{k dq \hat{r}_{ip}}{|\vec{r}_{ip}|^2}$$

$$\vec{r}_i = x\hat{x} + 0\hat{y} : \vec{r}_p = 0\hat{x} + y_p\hat{y} : \vec{r}_{ip} = \vec{r}_p - \vec{r}_i = -x\hat{x} + y_p\hat{y} : \hat{r}_{ip} = \frac{-x\hat{x} + y_p\hat{y}}{\sqrt{x^2 + y_p^2}}$$

$$dq = -\lambda dx [-a < x < 0] : dq = \lambda dx [0 < x < a]$$

$$\vec{E}_p = \int_{x=-a}^{x=0} -\frac{k\lambda(-x\hat{x} + y_p\hat{y})}{[x^2 + y_p^2]^{3/2}} dx + \int_{x=0}^{x=a} \frac{k\lambda(-x\hat{x} + y_p\hat{y})}{[x^2 + y_p^2]^{3/2}} dx$$

By symmetry (you can also show by doing the integral) the  $y$  component must vanish.

Thus:

$$\begin{aligned} \vec{E}_p &= k\lambda \left[ \int_{x=-a}^{x=0} \frac{x\hat{x}}{[x^2 + y_p^2]^{3/2}} dx - \int_{x=0}^{x=a} \frac{x\hat{x}}{[x^2 + y_p^2]^{3/2}} dx \right] \hat{x} \\ &= k\lambda \left[ -\frac{1}{\sqrt{x^2 + y_p^2}} \Big|_{-a}^0 + \frac{1}{\sqrt{x^2 + y_p^2}} \Big|_0^a \right] \hat{x} \\ &= k\lambda \left[ -\frac{1}{|y_p|} - \frac{1}{\sqrt{a^2 + y_p^2}} + \frac{1}{\sqrt{a^2 + y_p^2}} - \frac{1}{|y_p|} \right] \hat{x} \\ &= k\lambda \left[ -\frac{1}{|y_p|} + \frac{1}{\sqrt{a^2 + y_p^2}} + \frac{1}{\sqrt{a^2 + y_p^2}} - \frac{1}{|y_p|} \right] \hat{x} = 2k\lambda \left[ \frac{1}{\sqrt{a^2 + y_p^2}} - \frac{1}{|y_p|} \right] \hat{x} \end{aligned}$$

$$\int \frac{x}{(x^2 + y^2)^{3/2}} dx =$$

$$-\frac{1}{\sqrt{x^2 + y^2}}$$