1.54

$\vec{v} = ay\hat{x} + bx\hat{y}$

Stokes' Theorem: $\int_{\text{surface}} (\vec{\nabla}\times\vec{V})\bullet d\vec{A} = \oint_{\text{path}} \vec{V}\bullet d\vec{l}$

Circular path of radius R centered at the origin.

$$\vec{\nabla}\times\vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ay & bx & 0 \end{vmatrix} = \hat{x}(0) - \hat{y}(0) + \hat{z}(b-a)$$

$d\vec{A} = sdsd\varphi\hat{z}$

$$\Rightarrow \int_{\text{disk}} [\vec{\nabla}\times\vec{v}]\bullet d\vec{A} = \int_{s=0}^{s=R}\int_{\varphi=0}^{\varphi=2\pi}(b-a)sdsd\varphi = \tfrac{1}{2}R^2 2\pi(b-a) = \pi R^2(b-a)$$

On path: $d\vec{l} = dx\hat{x} + dy\hat{y}$

Convert to polar coordinates:
$$x = s\cos\varphi \Rightarrow dx = -s\sin\varphi d\varphi$$
$$y = s\sin\varphi \Rightarrow dy = s\cos\varphi d\varphi$$
$$\Rightarrow d\vec{l} = -s\sin\varphi d\varphi\hat{x} + s\cos\varphi d\varphi\hat{y}$$

$$\vec{v}\bullet d\vec{l} = [ay\hat{x} + bx\hat{y}]\bullet[-s\sin\varphi d\varphi\hat{x} + s\cos\varphi d\varphi\hat{y}]$$
$$= -say\sin\varphi d\varphi + sbx\cos\varphi d\varphi$$
$$= [-sa(s\sin\varphi)\sin\varphi + sb(s\cos\varphi)\cos\varphi]d\varphi$$
$$= [-as^2\sin^2\varphi + bs^2\cos^2\varphi]d\varphi$$
$$= [-as^2[1-\cos^2\varphi] + bs^2\cos^2\varphi]d\varphi$$
$$= [-as^2 + s^2(a+b)\cos^2\varphi]d\varphi$$
$$\Rightarrow \int_{\varphi=0}^{2\pi}[-as^2 + s^2(a+b)\cos^2\varphi]d\varphi =$$
$$= -as^2[2\pi] + s^2(a+b)\tfrac{1}{2}[\varphi + \cos\varphi\sin\varphi]\Big|_0^{2\pi} =$$
$$= -as^2[2\pi] + s^2(a+b)\tfrac{1}{2}2\pi = 2\pi s^2[-a + \tfrac{a}{2} + \tfrac{b}{2}] = \pi s^2(b-a)$$

Thus Stokes' theorem is verified.