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(1) Cowboy Justin is passing by the Rockin' R Bar* in Billings, Montana. There is a whole lot of craziness going on in the bar. In the heat of the moment, one of the bar patrons lets out a Hoop! And then a gun is pulled the wall is shot with a lead bullet which initially had a velocity v=120 m/s. If the specific heat of lead is 128 J/(kg ^{0}C), what is the change in temperature of the lead if the entire kinetic energy of the lead is converted into heat?

* A dairy bar

(2) Alcohol has a specific heat of 2400 J/(kg ^{0}C) and water has a specific heat of 4186 J/(kg ^{0}C). Suppose 1 kg of alcohol at 80 ^{0}C is mixed with 0.25 kg of water at 30 ^{0}C. What is the final equilibrium temperature of the mixture?

(3) Suppose an flute (assume mixed boundary conditions here) is made of copper (c_{cu}=387 J/(kg ^{0}C)), (α_{cu}=17.00x10^{-6} /^{0}C). The flute is observed to resonate at a frequency of 550.0 Hz in its fundamental mode of oscillation in a room which is at 30.00^{0}C. If the flute is taken outside where the temperature is at 0.00^{0}C, what is the frequency of oscillation of the flute? You may assume in both cases (although incorrectly) that the speed of sound is 343.0 m/s.

(4) Aluminum has a coefficient of linear expansion of 24x10^{-6} /^{0}C and the specific heat of aluminum is 900 J/(kg ^{0}C). Suppose a block of aluminum of 10 cm on a side has a density of 2.7x10^3 kg/m^3. We are going to expand this aluminum in a vacuum for reasons that will become clear in later lectures. The aluminum is heated from a temperature of 0^{0}C to a temperature of 100^{0}C. How much heat is added to the system? How much is the final volume of the block?

Thermodynamic work is defined by W=PΔV. If the bar were expanded against atmospheric pressure, how much work did the bar do in the expansion?

(5) Show how one can measure the specific heat of an unknown sample.

Heat and Thermodynamics

We will see later that temperature is a direct measure of average molecular kinetic energy. For now, however, we will assume that temperature is a measurable property of a substance that is somehow connected to heat.

A famous experiment done by James Joule reproducibly and clearly showed that there is a direct connection between the change in total mechanical energy and the addition of heat to a substance. I've constructed an animated gif which illustrates this classical experiment.

Linear Expansion

One of the first properties that one observes with regard to thermodynamics is that when substances are heated, they change length. Most substances, but not all, expand when their temperature increases. If, for example, we were to consider length as our bulk property, then to first order, the equation which describes this expansion is:

$$\Delta L = L_0 \alpha (\Delta T) \quad .$$

Here, ΔL is the change in length, L_0 is the length before the temperature change, ΔT is the temperature change and α is known as the coefficient of linear temperature expansion. Now, we can also talk about area expansion and volume expansion. I'll show below that each of these higher-dimensional coefficients is related to α for homogenous solids.

Area Expansion

Let's assume we have a homogenous solid (meaning, as far as expansion goes, everything is the same in all directions) in the shape of a thin plate of length L and width w. If we apply a change in temperature to a system with an initial area A_0, the change in area is given by:

$$\Delta A = A_0 \beta (\Delta T) \quad .$$

Let me now show the connection between β and α.

$$A_0 = LW$$

Let's apply a temperature change to this plate, (ΔT). Then,

$$\Delta L = L - L_0 = L_0 \alpha (\Delta T) \Rightarrow L = L_0 [1 + \alpha (\Delta T)]$$

$$\Delta W = W - W_0 = W_0 \alpha (\Delta T) \Rightarrow W = W_0 [1 + \alpha (\Delta T)]$$

The changed area is given by:

$$A = LW = L_0 W_0 [1 + \alpha (\Delta T)]^2 = A_0 \left[1 + 2\alpha (\Delta T) + (\alpha (\Delta T))^2\right]$$

The term $[\alpha (\Delta T)]^2 \approx 0$

by comparison to the other terms for relatively small temperature changes. Thus,

$$\Delta A = A_0 \beta \Delta T ; \beta = 2\alpha \quad .$$

Volume Expansion

Likewise, it is now straightforward to show, in the same manner that

$$\gamma = 3\alpha$$

here is how: consider an isotropic cube. Then each direction has the same coefficient of linear expansion. Upon expansion, the volume is given by:

$$V = [L_0 + L_0 \alpha \Delta T]^3 = L_0^3 [1 + \alpha \Delta T]^3 = V_0 \left[1 + 3\alpha(\Delta T) + 3\alpha^2 [\Delta T]^2 + \alpha^3 (\Delta T)^3\right]$$

Now if you ignore terms higher than α^1 , you then have:

$$V \approx V_0 [1 + 3\alpha \Delta T]$$

Identify now $\gamma = 3\alpha$ and you have:

$$V = V_0 \gamma \Delta T$$

Also, please note that these coefficients can be negative (rubber has this).

Heat and Calorimetry

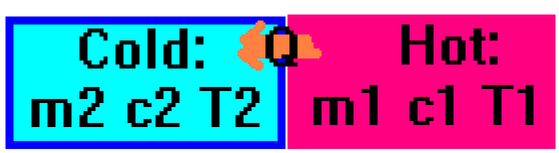
The fundamental idea to keep in mind when talking about heat is to remember that it is a form of energy (this was shown by Joule's experiment). We represent heat by the symbol Q and it has the same SI units as energy. When heat is applied to a body, it is observed that the temperature of the body increases. It is further observed that heat energy is transferred from a hotter object to a colder object. One final observation is that Heat, (Q) is not like kinetic energy … you can add a small amount of heat but

you can not have a change in Q.

Right: Q =… Wrong: $\cancel{\Delta}$Q =…

Let's see how two bodies exchange heat energy.

Place two bodies in thermal contact. Body 1 has mass m_1, specific heat c_1 and is initially at a temperature T_1. Body 2 has mass m_2, specific heat c_2 and is initially at a temperature T_2. The bodies exchange energy in the form of heat, Q. What is the final equilibrium temperature of the system? Note that equilibrium is established when no net energy exchange occurs between the two bodies and thus the two bodies have the same temperature.



The starting point to answer a question such as this is the conservation of energy. Thus,

$$\text{Energy is conserved} \Rightarrow Q = 0 \quad .$$

For each body, we find Q from:

$$Q_i = m_i c_i (\Delta T_i) \text{ and } Q = \sum_i Q_i$$

Thus, we have:

$$Q_1 + Q_2 = m_1 c_1 (T_f - T_1) + m_2 c_2 (T_f - T_2) = 0$$

This is fairly easily solved for T_f:

$$T_f = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}$$

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For calculus students:

What do you do if the specific heat of a substance obeys some function of temperature, such as, for example

$$c(T) = AT + BT^2 \quad ?$$

In this case, we obtain the heat added to a system from:

$$Q = \int_{T_{initial}}^{T_{final}} m\,c(T)\,dT$$

In the present example, we would have:

$$Q = m \int_{T_{initial}}^{T_{final}} \left(AT + BT^2\right) dT = m\left[\left[\frac{AT^2}{2}\right]_{t_{initial}}^{T^{final}} + \left[\frac{BT^3}{3}\right]_{T_{omotoa}}^{T_{final}};\right]$$

We can solve this for Q as:

$$Q = m\left[\frac{A}{2}\left(T_{final}^2 - T_{initial}^2\right) + \frac{B}{3}\left(T_{final}^3 - T_{initial}^3\right)\right]$$

Here's another example:

Suppose a system has a specific heat that varies exponentially over a certain range of temperature as:

$$C(T) = A e^{\frac{T}{B}}$$

The amount of heat required to change a mass of this material from an initial temperature to a final temperature is then given by:

$$Q = mA \int_{T_{initial}}^{T_{final}} e^{\frac{T}{B}} dT = mAB\left[e^{\frac{T}{B}}\right]_{T_{initial}}^{T_{final}} = mAB\left(e^{\frac{T_{final}}{B}} - e^{\frac{T_{initial}}{B}}\right]$$

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Solution: The initial kinetic energy of the bullet is given by:

$$K=\frac{1}{2}mv^2 \quad .$$

Energy conservation (assuming all this kinetic energy goes into heat energy) is expressed as:

$$Q+\Delta K=0 \quad .$$

We have then:

$$m c_{lead}(\Delta T)=\frac{1}{2}mv^2=0$$

We can solve this for the change in temperature:

$$\Delta T=\frac{v^2}{2c_{lead}}=\frac{\left(120\frac{m}{s}\right)^2}{2\left(128\frac{J}{kg\,^0C}\right)}=56.3\,^0C$$

(2) Alcohol has a specific heat of 2400 J/(kg ^{0}C) and water has a specific heat of 4186 J/(kg ^{0}C). Suppose 1 kg of alcohol at 80 ^{0}C is mixed with 0.25 kg of water at 30 ^{0}C. What is the final equilibrium temperature of the mixture?

Solution:
Energy is conserved. Thus, $Q=0$.
We thus have:

$$m_{alcohol}c_{alcohol}(\Delta T_{alcohol})+m_{water}c_{water}(\Delta T_{water})=0$$

We can solve this for the final equilibrium temperature:

Substituting in the given values, we then find:

$$T_f=\frac{m_{alcohol}c_{alcohol}T_{alcohol}+m_{water}c_{water}T_{water}}{m_{alcohol}c_{alcohol}+m_{water}c_{water}}$$

$$T_f=\frac{1.00(2400)80+0.25(4186)30}{1.00(2400)+0.25(4186)}=\frac{192000+31395}{2400+1046.5}=\frac{223395}{3446.5}=64.8\,^0C$$

(3) Suppose an flute (assume mixed boundary conditions here) is made of copper (c_{cu}=387 J/(kg ^{0}C)), (α_{cu}=17.00x10^{-6} /^{0}C). The flute is observed to resonate at a frequency of 550.0 Hz in its fundamental mode of oscillation in a room which is at 30.00^{0}C. If the flute is taken outside where the temperature is at 0.00^{0}C, what is the frequency of oscillation of the flute? You may assume in both cases (although incorrectly) that the speed of sound is 343.0 m/s.

Solution:
When the flute is playing inside, it has a length given by:

$$f_1 = \frac{v}{4L} \Rightarrow L = \frac{v}{4f_1} = \frac{343}{4(550)} = 0.1559\,m$$

Now, this length will change as:

$$\Delta L = L_0 \alpha(\Delta T) = 0.1559(17\,x\,10^{-6})(-30) = -7.951\,x\,10^{-5}\,m$$

The final length of the flute will be:

$$L = L_0 + \Delta L = 0.1559 - 7.951\,x\,10^{-5} = 0.1558\,m$$

The frequency outside will be:

$$f_1 = \frac{343}{4(0.1558)} = 550.4\,Hz$$

For people with very good ears, this might be just barely detectable.

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Thermodynamic work is defined by W=PΔV. If the bar were expanded against atmospheric pressure, how much work did the bar do in the expansion?

Solution:
The amount of heat added is given by:

$$Q = m_{al} c_{al} (\Delta T)$$

We're not given the mass directly here but it is given by:

$$m_{al} = \rho_{al} V_{al} = \left(2.7\,x\,10^3 \frac{kg}{m^3}\right)(0.1\,m)^3 = 2.7\,kg$$

The amount of heat added to the system is then given by:

$$Q = (2.7\,kg)\left(900\frac{J}{kg\,^0C}\right)(100\,^0C) = 2.43\,x\,10^5\,J$$

The change in volume is given by:

$$\Delta V = V_0(3\alpha)(\Delta T) = (0.1)^3(3\,x\,24\,x\,10^{-6}\,m^3)(100) = 7.2\,x\,10^{-6}\,m^3$$

The final volume is then

$$V = V_0 + \Delta V = 1.0072 \times 10^{-3}\, m^3$$

The thermodynamic work done in expanding against the atmosphere is:

$$W = P(\Delta V) = 1.013 \times 10^5\, Pa\left(7.2 \times 10^{-6} m^3\right) = 0.73\, J$$

Keep this result in mind for a later lecture.

(5) Show how one can measure the specific heat of an unknown sample.

Solution:
Actual calorimetry experiments require at least 3 masses, usually, the mass of water, the mass of an unknown and the mass of a cup used to do the calorimetry experiment. Let's see how to do this more complex calculation.

Suppose we define the following quantities:

Cup	Water	unknown
m_c	m_w	m_u
T_c	T_w	T_u
c_c	c_w	c_u

The idea here is that the unknown is heated to a certain temperature and then immersed into water. The final equilibrium temperature of the mixture is then measured.

The conservation of energy says:

$$Q=0$$

Thus:

$$m_c c_c(T-T_c)+m_w c_w(T-T_w)+m_u c_u(T-T_u)=0$$

Normally, we will have:

$$T-T_c=T-T_w$$

This helps us simplify the equation to read:

$$[m_c c_c+m_w c_w](T-T_w)+m_u c_u(T-T_u)=0$$

This is easily solved for the specific heat of the unknown:

$$c_u=-\frac{[m_c c_c+m_w c_w](T-T_w)}{m_u(T-T_u)}$$

Consider the following example:

Cup	Water	unknown
m_c=0.05kg	m_w=0.5 kg	m_u=0.27kg
T_c=20 ^{0}C	T_w=20^{0}C	T_u=100^{0}C
c_c=900 J/(kg ^{0}C)	c_w=4186 J/(kg ^{0}C)	c_u=?????

Suppose that this experiment provided an equilibrium temperature of 55^{0}C. Find the specific heat of the unknown substance.

$$c_u=-\frac{[m_c c_c+m_w c_w](T-T_w)}{m_u(T-T_u)}=-\frac{[0.05(900)+0.5(4186)](55-20)}{0.27(55-100)}=-\frac{[45+2093](35)}{0.27(-45)}=6159\frac{J}{kg^0C}$$