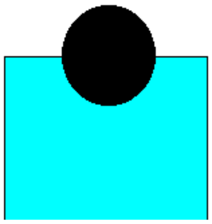
r20	Physics 240: Worksheet 26	Name: _______________



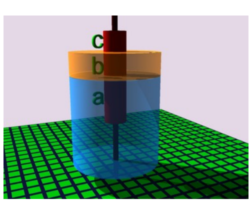
(1) A ball is floating in water. If the density of the ball is $\rho=0.95\text{x}10^3$ kg/m^3, what percentage of the ball is above the water?

(2) An object with a density of $\rho_{object}=19.3\text{x}10^3$ kg/m^3 and a mass m=50 kg is placed into a cylinder which contains a fluid.
(a) If the fluid is water, how much does the object weigh when it is in the water?
(b) Suppose the fluid has a density of $13\text{x}10^3$ kg/m^3. What is the apparent weight in this fluid?

(3) An object floats with 50% of its volume submerged in fluid 1 while it floats with 75% of its volume submerged in fluid 2. If fluid 1 is water, what is the density of fluid 2?

(4) The density of air is 1.29 kg/m^3. Gold has a density of $13\text{x}10^3$ kg/m^3. Suppose you have a piece of gold which weighs 1 kg. What is the radius of a sphere which could be made from gold which would float in air?

(5) Suppose helium has a density of $1.79\text{x}10^{-1}$ kg/m^3. A balloon with a volume V=0.5m^3 has a mass m=50g. When the balloon is filled with helium, how much additional weight will the balloon be capable of lifting?



(6) A cylindrical body of volume $V=AL:L=(a+b+c)$ and density $\rho=250$ kg/m^3 is floating in an oil-water mixture as shown with a rod to stabilize the cylinder. The oil layer has a thickness given by: $b=\eta L:0\le\eta\le1$. If the densities of the fluids are $\rho_{water}=1000$ kg/m^3 and $\rho_{oil}=800$ kg/m^3, what are the ratios

$\frac{a}{L}$ and $\frac{c}{L}$?

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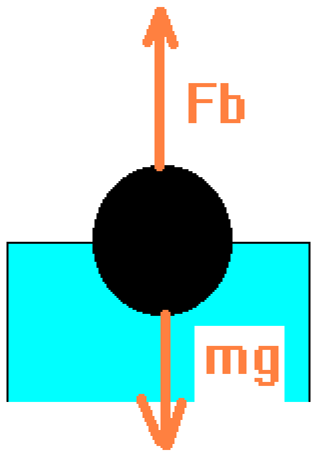

Solution: The principle involved here is Archimedes' principle which states that the **buoyant force is equal to the weight of the fluid displaced.** The object will float if the buoyant force is equal to the weight of the object.

Thus:

$$F_b = \text{ wt. of fluid displaced } = \rho_{water} V_{object_{submerged}} g$$

The second part is this: **the object floats if the buoyant force is equal to the weight of the object**.. Thus:

$$\text{wt. of object } = \rho_{object} V_{object} g$$

And since it is floating, we must then have:

$$\rho_{water} V_{object_{submerged}} g = \rho_{object} V_{object} g$$

We can thus find the fraction of the object submerged:

$$\frac{V_{object_{submerged}}}{V_{object}} = \frac{\rho_{object}}{\rho_{water}}$$

We are, however, asked for the percentage of the object above the water. This is given from:

$$V_{object} = V_{object_{submerged}} + V_{object_{not\ submerged}} \Rightarrow V_{object_{not\ submerged}} = V_{object} - V_{object_{submerged}}$$

Thus,

$$\frac{V_{object_{not\ submerged}}}{V_{object}} = \frac{V_{object}}{V_{object}} - \frac{V_{object_{submerged}}}{V_{object}} = 1 - \frac{\rho_{object}}{\rho_{water}}$$

The density of water is $\rho_{water} = 1 \text{x} 10^3 \frac{kg}{m^3}$ (***you should remember this value***). Thus we find:

$$\frac{V_{object_{not\ submerged}}}{V_{object}} = 1 - \frac{\rho_{object}}{\rho_{water}} = 1 - \frac{0.95 \text{x} 10^3}{1 \text{x} 10^3} = 0.05$$

This is the fraction of the ball which is above the water. In order to determine the percentage of the ball above the water, we multiply by 100 which gives 5% above the water.

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(2) An object with a density of ρ_{object}=19.3x10^3 kg/m^3 and a mass m=50 kg is placed into a cylinder which contains a fluid.
(a) If the fluid is water, how much does the object weigh when it is in the water?
(b) Suppose the fluid has a density of 13x10^3 kg/m^3. What is the apparent weight in this fluid?

Solution:
(a) Since the object has a density higher than that of water, the object will not float in water. Thus it sinks. When it sinks, it will still displace water and there will be a buoyant force on the object which decreases its weight below the weight that the object has in air. The change in weight is given by the buoyant force. Thus:

$$\Delta(\text{weight}) = F_b$$

We can find out the volume of the fluid displaced if we know the volume of the object. This is given by:

$$m = \rho V \Rightarrow V = \frac{m}{\rho} = \frac{50\,\text{kg}}{19.3 \times 10^3\,\text{kg/m}^3} = 2.59 \times 10^{-3}\,\text{m}^3$$

The change in weight is given by:

$$F_b = \rho_{fluid}\, g V = (1 \times 10^3) \times (9.8) \times (2.59 \times 10^3) = 25.4\,\text{N}$$

The weight of the object is given by mg. Thus:

$$\text{weight} = mg = (50) \times (9.8) = 490\,\text{N}$$

The apparent weight of the object in water is then 490N-25.4N=465N.
(b) This fluid is most likely something like mercury. We can repeat the calculation for this new fluid. The object will still sink in this fluid since the density of the object is greater than that of the fluid. The buoyant force is now given by:

$$F_b = \rho_{fluid}\, g V = (13 \times 10^3) \times (9.8) \times (2.59 \times 10^{-3}) = 330\,\text{N}$$

which is the change in apparent weight of the object. Thus the apparent weight of the object in this new fluid will be 490N-330N=160N.

The object has the same density as gold.

(3) An object floats with 50% of its volume submerged in fluid 1 while it floats with 75% of its volume submerged in fluid 2. If fluid 1 is water, what is the density of fluid 2?

Solution: As in previous problems, the principle involved here is Archimedes' principle. Thus:, for fluid 1:

$$F_b = \rho_{water} V_{object_{submerged}} g \quad .$$

The object floats with

$$\frac{V_{object_{submerged}}}{V_{object}} = 0.5$$

Thus, we can find the buoyant force at this point as:

$$F_b = 0.5\,\rho_{water} V_{object}\, g$$

Now since the object is floating, this is also equal to the weight of the object, which is given by:

$$weight = \rho_{object} V_{object}\, g$$

We then have:

$$0.5\,\rho_{water} V_{object}\, g = \rho_{object} V_{object}\, g \Rightarrow \rho_{object} = 0.5\,\rho_{water}$$

Now that we know the density of the object, we'll be able to find the density of fluid 2. We have that in fluid 2:

$$\frac{V_{object_{submerged}}}{V_{object}} = 0.75$$

Thus, the buoyant force at this point is given by:

$$F_b = 0.75\,\rho_{fluid\,2} V_{object}\, g$$

And since the object is floating, this will be equal to the weight of the object. But the weight of the object is given by

$$weight = \rho_{object} V_{object}\, g = 0.5\,\rho_{water} V_{object}\, g$$

Equating the two, we then find:

$$0.75\,\rho_{fluid2} V_{object}\, g = 0.5\,\rho_{water} V_{object}\, g \Rightarrow \rho_{fluid\,2} = \frac{0.5}{0.75}\rho_{water} = 667\,\frac{kg}{m^3}$$

(4) The density of air is 1.29 kg/m^3. Gold has a density of 19x10^3 kg/m^3. Suppose you have a piece of gold which weighs 1 kg. What is the radius of a sphere which could be made from this gold which would float in air?

Solution:
Although this problem may be technically impractical, let's see if it is. The type of sphere which would need to be made from the gold is one which weighs 1 kg, but is hollow and evacuated inside. Essentially, the average density of the sphere would have to be less than that of air. (This is the same idea that allows aircraft carriers to float on water). The density of the spherical object would thus need to be the same as that of air. The mass is given then by:

$$m_{object}=\rho_{air}V_{object}$$

We can thus find:

$$V_{object}=\frac{m_{object}}{\rho_{air}}=\frac{1\,kg}{1.29\,kg/m^3}=0.775\,m^3$$

Now, the volume of a sphere is given by

$$V_{sphere}=\frac{4}{3}\pi R^3 \Rightarrow R=\sqrt[3]{\frac{3V}{4\pi}}=\sqrt[3]{\frac{3(0.775)}{4\pi}}=0.57\,m$$

The interior of the sphere needs to be a vacuum.

The rest of this problem is for your interest only.

We can get an idea of how thin the gold shell would be for this sphere. The equation we need involves subtracting the volume of two spheres and equating this difference to the volume of gold involved. It's probably easier to see the math here than to interpret the words.

The volume of our gold is given by:

$$V_{gold}=\frac{m_{gold}}{\rho_{gold}}=\frac{1\,kg}{13\,x\,10^3\,kg;/m^3}=7.69\,x\,10^{-5}\,m^3$$

This volume of gold needs to be squished into a spherical shell of outer radius R_1 and inner radius R_2 where R_1 we've already found to be 0.57m.

Thus:

$$\frac{4}{3}\pi\left(R_1^3-R_2^3\right)=V_{gold}$$

We'll solve this for R_2:

$$R_2=\sqrt[3]{R_1^3-\frac{3}{4\pi}V_{gold}}=\sqrt[3]{0.57^3-\frac{3}{4\pi}7.69\,x\,10^{-5}}=0.56980\,m$$

A more precise value for R_1 is: 0.56982m The thickness of the gold film would be: 0.56982-0.56980=2x10^{-5}m. This film would need to hold back the enormous pressure of the Earth's atmosphere. Gold won't stand up to this type of pressure at this thickness so the sphere probably could not be made of gold. In fact, I am not aware of any material which would work here.

(5) Suppose helium has a density of $\rho_{HE}=1.79\,x\,10^{-1}\frac{kg}{m^3}$. A balloon with a volume $V=0.5\,m^3$ has a mass m=50g. When the balloon is filled with helium, how much additional weight will the balloon be capable of lifting?

Solution:
The buoyant force is given by the weight of the fluid displaced. In this problem, the displaced fluid is air with a density of 1.29 kg/m^3. The balloon has a mass of 50g or 0.05 kg. 0.5 m^3 of helium has a mass of

$$m_{helium}=\rho_{helium}V_{balloon}=\left(1.79\,x\,10^{-1}\frac{kg}{m^3}\right)\left(0.5\,m^3\right)=0.0895\,kg$$

The buoyant force is given by:

$$F_b=\rho_{air}V_{balloon}g=\left(1.29\frac{kg}{m^3}\right)\left(0.5\,m^3\right)\left(9.8\frac{m}{s^2}\right)=6.321\,N$$

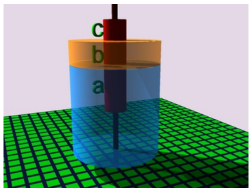
The weight of the balloon and helium is then:

$$W_{b+h}=(0.05+0.0895)9.98=1.367\,N$$

The balloon can thus lift an additional weight of

$$lift=6.32\,N-1.37\,N=4.59\,N$$

This is about ½ kg. The value is probably larger than what will actually happen because helium in the balloon may not be pure. You might, however, want to do the same problem with hydrogen (ρ=8.99x10^{-2} kg/m^3) and think about the Hindenburg. In any event, this does seem like quite a bit of lift.



(6) A cylindrical body of volume $V=AL:L=(a+b+c)$ and density $\rho=250$ kg/m^3 is floating in an oil-water mixture as shown with a rod to stabilize the cylinder. The oil layer has a thickness given by: $b=\eta L:0\leq\eta\leq 1$. If the densities of the fluids are $\rho_{water}=1000$ kg/m^3 and $\rho_{oil}=800$ kg/m^3, what are the ratios $\frac{a}{L}$ and $\frac{c}{L}$?

Solution: According to Archimedes' principle the weight of the fluid displaced is equal to the buoyant force. Let's assume the cylinder has a constant cross sectional area A. Then the buoyant force is given by:

$$F_b=aA\rho_{water}g+bA\rho_{oil}g=A[a\rho_{water}+b\rho_{oil}]g$$

Since the object is floating, the buoyant force is equal to the weight of the object. Thus:

$$F_b=\rho_{object}ALg$$

The two forces are equal. Thus:

$$A[a\rho_{water}+b\rho_{oil}]g=\rho_{object}ALg\Rightarrow[a\rho_{water}+b\rho_{oil}]=\rho_{object}L$$

Now we know how this is related to the thickness of the oil layer (look at the wording of the problem to see this). Thus:

$$[a\rho_{water}+b\rho_{oil}]=\rho_{object}\left[\frac{b}{\eta}\right]$$

It is now easy to solve this for a/L:

$$a\rho_{water}=\rho_{object}\left[\frac{b}{\eta}\right]-b\rho_{oil}=b\left[\frac{\rho_{object}}{\eta}-\rho_{oil}\right]=\eta L\left[\frac{\rho_{object}}{\eta}-\rho_{oil}\right]\Rightarrow\frac{a}{L}=\frac{\rho_{object}}{\rho_{water}}-\eta\frac{\rho_{oil}}{\rho_{water}}$$

It is also easy now to solve for c by replacing a in this expressing by L-b-c:

$$\frac{L-b-c}{L}=1-\frac{b}{L}-\frac{c}{L}=1-\eta-\frac{c}{L}=\frac{\rho_{object}}{\rho_{water}}-\eta\frac{\rho_{oil}}{\rho_{waer}}\Rightarrow\frac{c}{L}=1-\frac{\rho_{object}}{\rho_{waer}}+\eta\left[\frac{\rho_{oil}}{\rho_{water}}-1\right]$$

It's reasonable to check these answers against: $\frac{a+b+c}{L}=1$.

$$\frac{a}{L}+\frac{b}{L}+\frac{c}{L}=\frac{\rho_{object}}{\rho_{water}}-\eta\frac{\rho_{oil}}{\rho_{waer}}+\eta+1-\frac{\rho_{object}}{\rho_{water}}+\eta\left[\frac{\rho_{oil}}{\rho_{water}}-1\right]=1$$

For the particular problem at hand, we then have:

$$\frac{a}{L}=\frac{\rho_{object}}{\rho_{water}}-\eta\frac{\rho_{oil}}{\rho_{water}}=0.025-0.80\eta:\frac{c}{L}=0.75+0.20\eta$$

As a limiting cases: suppose

$$\eta=0\Rightarrow\frac{a}{L}=0.25:\frac{c}{L}=0.75$$.

Notice that there is a certain point at which this formulation of the problem will not work because I have assumed that the object displaces an amount of oil equal in volume to: ηLA. The smallest value of a for which this problem will work is then given by:

$$\frac{a}{L}=0 \Rightarrow 0=\frac{\rho_{object}}{\rho_{water}}-\eta\frac{\rho_{oil}}{\rho_{water}} \Rightarrow \frac{\rho_{object}}{\rho_{water}}=\eta\frac{\rho_{oil}}{\rho_{waer}} \Rightarrow \eta=\frac{\rho_{object}}{\rho_{oil}}=\frac{250}{800}=0.3125$$

After this point, the object floats completely in the oil regardless of how much deeper the oil layer goes. In fact, this is exactly the ratio b/L that would be obtained if the object floated only in oil.