| r20 | Physics 240: Worksheet 25 | Name __________________ |
|---|---|---|

(1) Suppose Seth, Talli and Ryan decided to take up the extreme sport of sky diving. The three go up in an airplane and Seth jumps first (after being scared by a cat), and then Ryan. Talli then changed her mind about the whole thing and waved goodbye. Seth and Ryan quickly reached a terminal speed of about 53 m/s. Suppose that Seth was screaming at a frequency of 440 Hz on the way down. What did each person hear for the frequency of Seth? Assume the speed of sound is 343 m/s.

(2) Suppose an open organ pipe of length $L_1$=1 m is in its first mode of oscillation. A second organ pipe, which is longer is also in its first mode of oscillation. An observer hears a beat frequency of 3 Hz. How much longer is the second organ pipe? The speed of sound is 343 m/s.

(3) Suppose that you are driving your car ($v_o$=30 m/s) along side a train.($v_s$=10 m/s ) and the train blows its horn (f=600 Hz). What do you hear for the frequency of the horn? Next, suppose you pass the train while the horn is still blowing. What frequency do you hear then?

(4) Explain how a sonic boom occurs from the Doppler Shift. Can this happen for light?

(5) Two piano strings of the same length are vibrating in the second mode of oscillation. The tension in one string is 0.1N more than the first which had a tension of 1.5N. What is the beat frequency which is observed if the first string has a frequency of 660Hz?

**Sound: beats and the Doppler shift**

Beats will occur whenever two traveling waves have a difference in frequencies. Let's see how this occurs:

Suppose the two traveling waves are given by:

$$s_1 = A\sin(k_1 x - \omega_1 t) \quad \text{and} \quad s_2 = A\sin(k_2 x - \omega_2 t)$$

In this case, of course, we require that

$$v = \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{k_1}{k_2}$$

so that the waves are traveling at the same velocity.
Let's look at the superposition of these two waves:

$$S = s_1 + s_2 = A\left[\sin(k_1 x - \omega_1 t) + \sin(k_2 x - \omega_2 t)\right]$$

A very useful trig identity:

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{1}{2}(\alpha+\beta)\right)\cos\left(\frac{1}{2}(\alpha-\beta)\right)$$

Thus, the superposition of these two waves will be:

$$S = 2A\sin\left(\frac{1}{2}\left[(k_1+k_2)x - (\omega_1+\omega_2)t\right]\right)\cos\left(\frac{1}{2}\left[(k_1-k_2)x - (\omega_1-\omega_2)t\right]\right)$$

Now, like it or not, this is exactly the result that you will get if you add two traveling waves together. We can make the analysis a bit easier by choosing x=0 for our ear location ... but we don't really need to do this at all. For our ears to perceive beats, instead of two separate sounds, we need that the two frequencies be fairly close together. This means that the ratio of the frequencies is about equal to 1. Now it is time to locate your ear at x=0. What results is:

$$S = 2A\sin\left(\frac{1}{2}\left[-(\omega_1+\omega_2)t\right]\right)\cos\left(\frac{1}{2}\left[-(\omega_1-\omega_2)t\right]\right)$$

This superposition clearly shows 2 waveforms ... let's label these two frequencies:

$$\omega_h = \omega_1 + \omega_2 \quad \text{and} \quad \omega_L = \omega_1 - \omega_2$$

With this, the waveform appears as:

$$S = 2A\sin\left(\frac{1}{2}\left[-\omega_h t\right]\right)\cos\left(\frac{1}{2}\left[-\omega_L t\right]\right)$$

The high frequency wave (as refers to sound) will probably not be detectable although in certain experiments on systems other than sound, instruments may be capable of detecting this frequency. The low frequency component is, however, of interest because it leads to the overall "packet" modulation at a very low frequency which the human ear can detect.

Here, h stands for “high” and L stands for “low.”

It is interesting to note, however, that this low frequency oscillation will be detected at a frequency given by the difference in the actual frequencies which is called the "beat frequency":

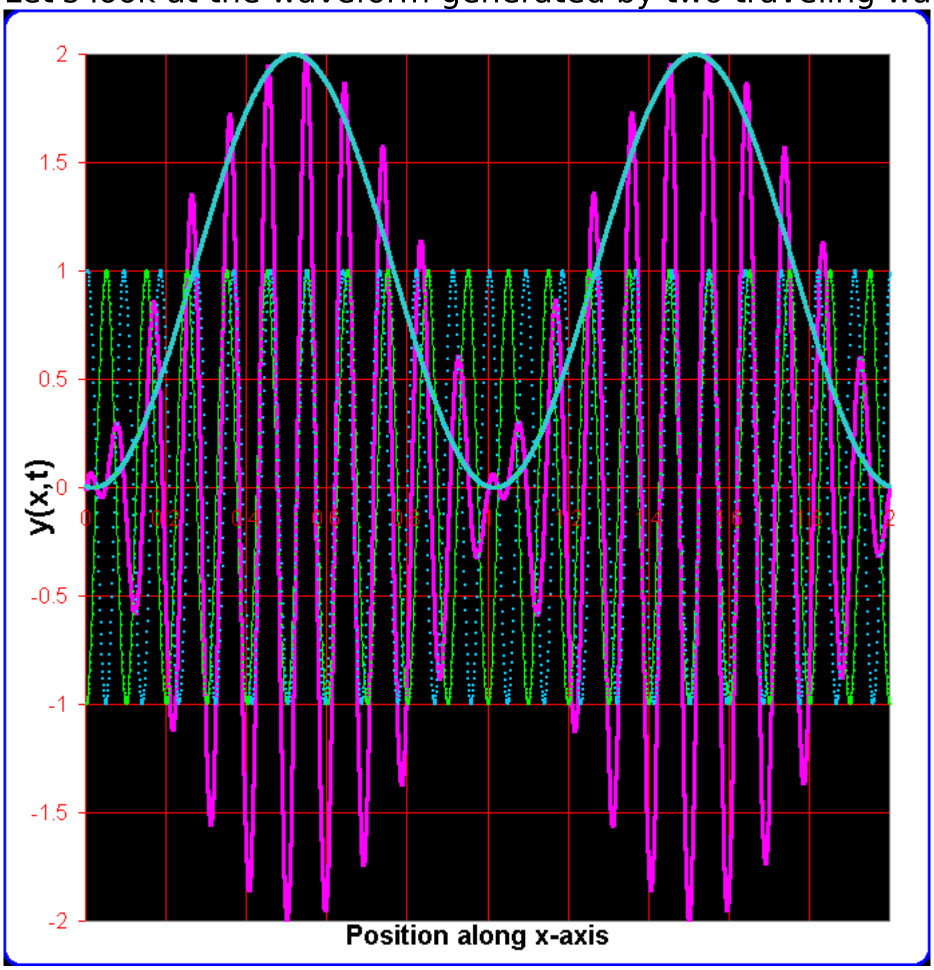
**Right beat frequency!--→** $f_b = |f_1 - f_2|$ **and** $\omega_b = \omega_L$

The absolute value is because cos(-x)=cos(x) so it doesn't matter which is named 1 and 2.

**Now the final detail: Why is the beat frequency not given by**

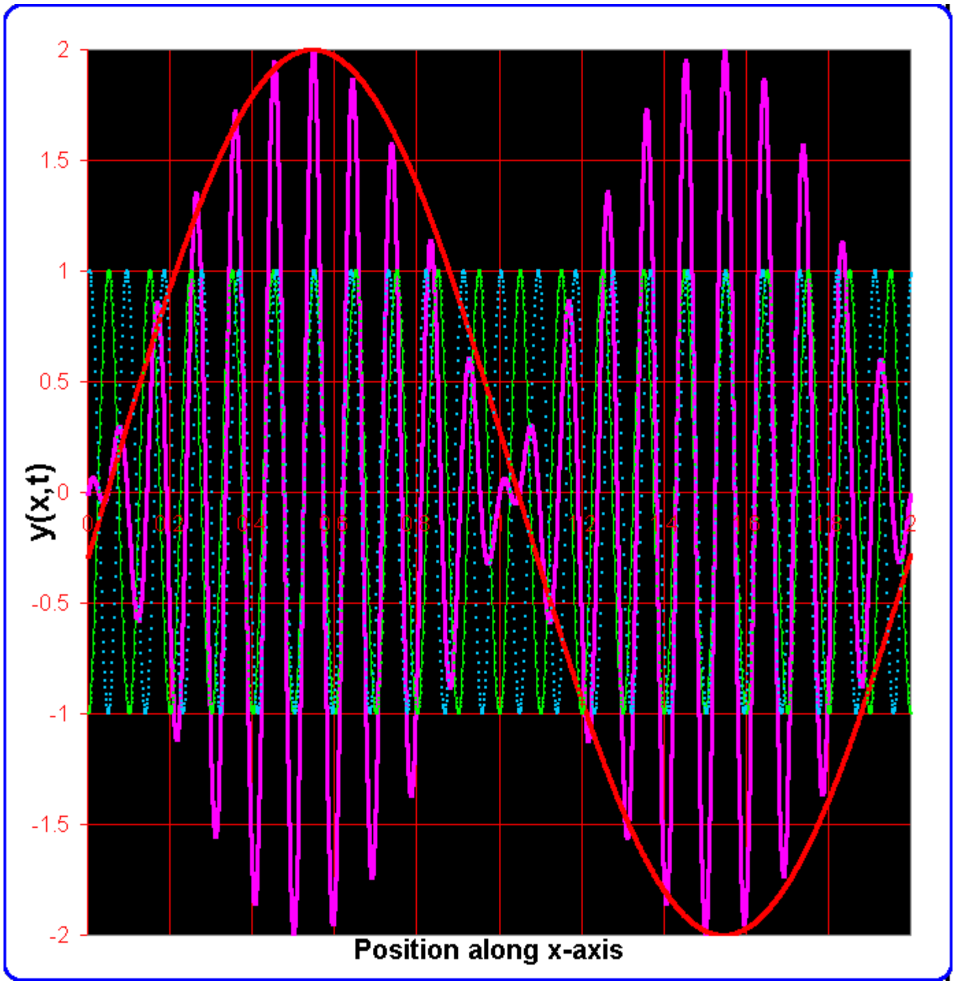
$f_{b_{wronganswer}} = \frac{1}{2}|f_1 - f_2|$ **<--- Wrong beat frequency!**

Let's look at the waveform generated by two traveling waves:




What I have plotted is a phase-shifted cosine curve (blue) which has a frequency of oscillation equal to the beat frequency $f_b$. Since it is the envelope of the beats that you would hear, this is indeed the beat frequency and not the "wrong" answer of 1/2 the beat frequency. **Your ear would detect 2 beats in the picture shown above with the envelope correctly described with a beat frequency of $f_b$.**

Essentially, if you (incorrectly) thought that the beat frequency was 1/2 of what it actually is, you would have the "red" envelope which I show below. Everything has been duplicated, hence the **incorrect** factor of 2.



**The Doppler shift**

**I have a song about the Doppler shift for you to listen to.**

The Doppler shift comes about for sound due to 2 factors ...
(1) The sound source moves with a speed $v_{source}=v_s$
This implies a shorter or longer wavelength for the sound
and
(2) The observer moves with a speed $v_{observer}=v_o$
This implies a higher or lower measured speed of sound

All of this works pretty well so long as our waves are sound waves. If, however, you try to talk about light waves, since each observer measures the same speed for the speed of light, you won't see the second effect at all.
In the following, let v be the speed of sound.
We also let the ' notation refer to what the observer detects.

---

Let's first let $v_{observer}=v_o=0$.

When the source moves towards the observer, the wavelength of the emitted sound is shortened. We can determine how much pretty easily ...

$$\lambda_{\text{stationary source}}=\frac{v}{f}$$

In one period (T), for a moving source, the source moves through a distance:

$$\delta x=v_x T=\frac{v_s}{f}$$

If the source moves toward the observer, then the wavelength will be shortened by δx.

Thus, $\lambda'=\lambda_{\text{stationary source}}-\delta x=\frac{v}{f}-\frac{v_s}{f}=\frac{1}{f}(v-v_s)$ .

We ultimately want to write this as

$$f'\lambda'=v$$

The observer would detect then:

$$f'=\frac{v}{\lambda'}=\frac{v}{\left(\frac{1}{f}(v-v_s)\right)}=f\frac{v}{v-v_s}=f\frac{1}{1-\frac{v_s}{v}}$$

If the source moved away from the observer, the corresponding result is:

$$f'=\frac{v}{\lambda'}=\frac{v}{\left(\frac{1}{f}(v+v_s)\right)}=f\frac{v}{v+v_s}=f\frac{1}{1+\frac{v_s}{v}}\ .$$

We combine these two results into one expression:

$$f'=f\frac{1}{1\mp\frac{v_s}{v}}$$

where the top sign refers to "towards" and the bottom sign refers to "away".

Now, let's let the observer move towards a stationary source.
In this case, the speed of sound as measured by the observer would be higher than an observer at rest with respect to the incoming sound waves. Thus:

$$v' = v + v_0$$

We want to write this again in terms of $f'\lambda' = v$ or:

$$f' = \frac{v+v_0}{\lambda} = \frac{v}{\lambda}\left(1+\frac{v_0}{f}\right) = f\left(1+\frac{v_0}{v}\right)$$

Now if the observer was moving away from a stationary source, we would have:

$$f' = \frac{v-v_0}{\lambda} = \frac{v}{\lambda}\left(1-\frac{v_0}{f}\right) = f\left(1-\frac{v_0}{v}\right) \quad .$$

These can be combined into one compact result:

$$f' = f\left(1 \pm \frac{v_0}{v}\right)$$

where the top sign refers to "towards" and the bottom sign refers to "away".

In the general case of a moving observer and also a moving source, we then have:

$$f' = f\left[\frac{1 \pm \frac{v_0}{v}}{1 \mp \frac{v_s}{v}}\right] \quad .$$

Where the top sign refers to "towards" and the bottom sign refers to "away".

(1) Suppose Seth, Talli and Ryan decided to take up the extreme sport of sky diving. The three go up in an airplane and Seth jumps first (after being scared by a cat), and then Ryan. Talli then changed her mind about the whole thing and waved goodbye. Seth and Ryan quickly reached a terminal speed of about 53 m/s. Suppose that Seth was screaming at a frequency of 440 Hz on the way down. What did each person hear for the frequency of Seth? Assume the speed of sound is 343 m/s.

Solution:

Seth hears a frequency of 440 Hz.

For the others,

$$f' = f\left[\frac{1 \pm \frac{v_0}{v}}{1 \mp \frac{v_s}{v}}\right]$$

For Ryan: Ryan is falling towards Seth but Seth is falling away from Ryan just as fast. Thus:

$$f'_{ryan} = 440\left[\frac{1+\frac{53}{343}}{1+\frac{53}{343}}\right] = 440\,\text{Hz}$$

For Talli: Seth is falling away from Talli while Talli is staying put. Thus:

$$f'_{Talli} = 440\left[\frac{1}{1+\frac{53}{343}}\right] = 381\,\text{Hz}$$

(2) Suppose an open organ pipe of length $L_1$=1 m is in its first mode of oscillation. A second organ pipe, which is longer is also in its first mode of oscillation. An observer hears a beat frequency of 3 Hz. How much longer is the second organ pipe? The speed of sound is 343 m/s.
Solution:
For the first mode of oscillation,

$$f_1=\frac{v}{2L}=\frac{343}{2}=171.5\,\mathrm{Hz} \quad .$$

The second organ pipe is oscillating at a frequency of either 174.5 Hz or 168.5 Hz, which ever gives the longer length. The Length of the second organ pipe is given by:

$$L=\frac{v}{2f}=\frac{343}{2(168.5)}=1.02\,\mathrm{m} \quad .$$

The other frequency would give a shorter length than 1 m.

(3) Suppose that you are driving your car ($v_o$=30 m/s) along side a train.($v_s$=10 m/s ) and the train blows its horn (f=600 Hz). What do you hear for the frequency of the horn? Next, suppose you pass the train while the horn is still blowing. What frequency do you hear then?
Solution:
1st. case:
Here, the train is moving away from the car and the car is moving towards the train.

Thus:

$$f'=f\left[\frac{1\pm\frac{v_0}{v}}{1\mp\frac{v_s}{v}}\right]=600\left[\frac{1+\frac{30}{343}}{1+\frac{10}{343}}\right]=634\,\mathrm{Hz}\quad .$$

2nd. case:
Here, the train is moving towards the car and the car is moving away from the train.

$$f'=f\left[\frac{1\pm\frac{v_0}{v}}{1\mp\frac{v_s}{v}}\right]=600\left[\frac{1-\frac{30}{343}}{1-\frac{10}{343}}\right]=564\,\mathrm{Hz}\quad .$$

Thus:

(4) Explain how a sonic boom occurs from the Doppler Shift. Can this happen for light?
Solution:
Imagine that an observer is fixed and a source is moving towards the observer. Then:

$$\frac{f'}{f}=\frac{1}{1-\frac{v_s}{v}}\quad .$$

As the speed approaches the speed of sound, the Doppler frequency shift begins to approach infinity, which it can't really do. Instead, it booms. I'll show you the Thrust SSC movie here. For light traveling in a vacuum, this can not happen. However, if light is in a medium such as water and a particle inside the water approaches the speed of light (in water), then indeed radiation will be given off. This type of radiation is has a bluish color and is seen around nuclear reactors when subatomic particles exceed the speed of light in water.

(5) Two piano strings of the same length are vibrating in the second mode of oscillation. The tension in one string is 0.1N more than the first which had a tension of 1.5N. What is the beat frequency which is observed if the first string has a frequency of 660Hz?

Solution:

Since both string are vibrating in the 2nd mode, and we are considering fixed boundary conditions, we have:

$$^1f_2 = \frac{v}{L} = \frac{\sqrt{\frac{^1T}{\mu}}}{L} \quad \text{and} \quad ^2f_2 = \frac{v}{L} = \frac{\sqrt{\frac{^2T}{\mu}}}{L} \quad .$$

The ratio of these frequencies is then

$$\frac{^1f_2}{^2f_2} = \frac{\sqrt{\frac{^1T}{\mu}}}{\sqrt{\frac{^2T}{\mu}}} = \sqrt{\frac{^1T}{^2T}} = \sqrt{\frac{1.5}{1.6}} = 0.968$$

The beat frequency would be 682-660=22Hz. It most likely would be at the upper range to be detectable as beats.