

A mass m_1 collides and sticks to a mass m_2 . After the collision, the mass combination is seen to move with a speed v .

(a) If mass m_2 was at rest before the collision, how fast was mass m_1 moving before the collision in terms of m_1 , m_2 and v ?

(b) Suppose the mass combination encounters a frictionless inclined plane with a moderate angle of inclination. How far above the base of the plane will the mass combination be at the instant it stops in terms of v and g ?

(c) If m_1 acquired its original momentum from the decompression of a spring of spring constant k , how much was the spring compressed in terms of k , m_1 , m_2 and v ?

(d) Suppose a force $|\vec{F}|=4ct^3$ from $t=0$ to $t=4s$ was applied to the mass. What was the change in velocity of the mass if the mass was 8 kg and $c=1 \text{ N/s}^3$?

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(a) If mass m_2 was at rest before the collision, how fast was mass m_1 moving before the collision in terms of m_1 , m_2 and v ?

Assuming that the collision is totally inelastic, we have:

$$\Delta \vec{P} = \vec{0} \Rightarrow m_1 v_{1,b} + m_2 v_{2,b} = (m_1 + m_2) v_A : v_{2,b} = 0 \Rightarrow v_{1,b} = \frac{m_1 + m_2}{m_1} v$$

(b) What fraction of the initial kinetic energy was lost in this collision in terms of m_1 , m_2 and v ?

For a totally inelastic collision,

$$\begin{aligned} \text{fraction lost} &= 1 - \frac{K_f}{K_i} \\ &\Rightarrow 1 - \frac{\frac{1}{2}(m_1 + m_2)v^2}{\frac{1}{2}m_1 v_{1,b}^2} = 1 - \frac{(m_1 + m_2)v^2}{m_1 \left(\frac{m_1 + m_2}{m_1}\right)^2 v^2} = 1 - \frac{m_1}{m_1 + m_2} = \frac{m_1 + m_2}{m_1 + m_2} - \frac{m_1}{m_1 + m_2} \\ &\Rightarrow \text{fraction lost} = \frac{m_2}{m_1 + m_2} : \% \text{ lost} = 100 \times \frac{m_2}{m_1 + m_2} \end{aligned}$$

In particular, note that if $m_2 = 0$ then no kinetic energy is lost.

(c) If m_1 acquired its original momentum from the decompression of a spring of spring constant k , how much was the spring compressed in terms of k , m_1 , m_2 and v ?

$$\begin{aligned} \Delta K_{NC} &= \Delta K_C + \Delta U \\ \Delta K_{NC} = 0 : \Delta K_C &= \frac{1}{2} m_1 v_{1,b}^2 : \Delta U = \Delta U_s = -\frac{1}{2} k x^2 \Rightarrow 0 = \frac{1}{2} m_1 v_{1,b}^2 - \frac{1}{2} k x^2 \Rightarrow x = \sqrt{\frac{m_1}{k}} v_{1,b} \\ x &= \sqrt{\frac{m_1}{k}} \frac{(m_1 + m_2)}{m_1} v = \sqrt{\frac{(m_1 + m_2)^2}{k m_1}} v \end{aligned}$$

(d) Suppose a force $|\vec{F}| = 4ct^3$ from $t=0$ to $t=4s$ was applied to the mass. What was the change in velocity of the mass if the mass was 8 kg and $c=1 \text{ N/s}^3$?

$$\begin{aligned} \vec{J} &= \int \vec{F} dt = \Delta \vec{p} = m \Delta \vec{v} \Rightarrow \Delta \vec{v} = \frac{\int \vec{F} dt}{m} \\ \text{One dimension} \Rightarrow \Delta v &= \frac{\int_{t=0}^{t=4} (4)(1)t^3 dt}{8} = \frac{1}{8} [t^4]_0^4 = \frac{256}{8} = 32 \frac{\text{m}}{\text{s}} \end{aligned}$$