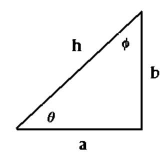
R20	Physics 240: Worksheet 01	Name ________________

(1) Which of the following equations are dimensionally correct? Note: the following quantities have the following dimensions:

$$F:\left[\frac{ML}{t^2}\right];m:[M];a:\left[\frac{L}{t^2}\right];t:[t];x[L];E:\left[\frac{ML^2}{t^2}\right];v:\left[\frac{L}{t}\right];s:[L]$$

(a) F=ma
(b) $X=(1/2)at^3$
(c) E=(1/2)mv
(d) E=max
(e) $V=[Fs/m]^{1/2}$

(2) In the right triangle shown, find the following: h in terms of a and b. Then find sin(θ), cos(θ) and tan(θ), sin(φ), cos(φ) and tan(φ).



(3) A vector $\vec{A}$ is given by $\vec{A}=5\hat{i}+4\hat{j}$. Find the following:
(a) what is the magnitude of $\vec{A}$?
(b) what is the angle the vector makes with the x-axis?
(c) what is the angle the vector makes with the y-axis?
(d) Express this vector using the "hat" notation.
(e) Express this vector using the "x-y" unit vector notation.

(4) Suppose a vector $\vec{B}$ is given by $\vec{B}=3\hat{i}+2\hat{j}$. Find the following:
(a)What is $2\vec{B}$?
(b) What is $\vec{B}+\vec{A}$?
(c) What is $\vec{B}-\vec{A}$?
(d) What is $\vec{B}\cdot\vec{A}$? (dot product)
(e) What is the angle made with respect to the positive x-axis by $\vec{B}+\vec{A}$?

(5) Suppose a vector $\vec{C}$ is given by $\vec{C}=8\hat{i}-9\hat{j}$. A person walks along vector $\vec{A}$, then vector $\vec{B}$, followed by vector $\vec{C}$. At the end of this journey, what is the displacement (vector) and the distance from the origin. You may assume all units are in m here.

(1) Which of the following equations are dimensionally correct? Note: the following quantities have the following dimensions:

$$F:\left[\frac{ML}{t^2}\right];m:[M];a:\left[\frac{L}{t^2}\right];t:[t];x[L];E:\left[\frac{ML^2}{t^2}\right];v:\left[\frac{L}{t}\right];s:[L]$$

(a) F=ma
(b) $X=(1/2)at^3$
(c) E=(1/2)mv
(d) E=max
(e) $v=\sqrt{\frac{Fs}{m}}$

Solution:

(a) yes: $F=\left[\frac{ML}{t^2}\right]$ as dimensions is correct. m= [M] and $a=\left[\frac{L}{t^2}\right]$,So you can see that $\left[\frac{ML}{t^2}\right]=[M]\left[\frac{L}{t^2}\right]$ which is in fact Newton's law $\vec{F}=m\vec{a}$.

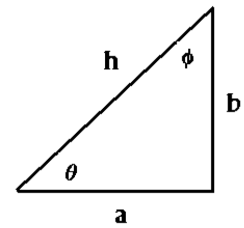
(b) no: $x=[L],a=\left[\frac{L}{t^2}\right]$ and $t=[t]$ so $[L]\neq[Lt]$

(c) no: $E=\left[\frac{ML^2}{t^2}\right]$ and $m=[M],v=\left[\frac{L}{t}\right]$ so $\left[\frac{ML^2}{t^2}\right]\neq\left[\frac{ML}{t}\right]$

(d) yes: $E=\left[M\frac{L^2}{t^2}\right],m=[M],a=\left[\frac{L}{t^2}\right],x=[L]$ so $\left[\frac{ML^2}{t^2}\right]=[M]\left[\frac{L}{t^2}\right]$

(e) yes: $v=\left[\frac{L}{t}\right],F=\left[\frac{ML}{t^2}\right],s=[L],m=[M]$ so $\left[\frac{L}{t}\right]=\left[\frac{L^2}{t^2}\right]^{1/2}$

(2) In the right triangle shown, find the following: h in terms of a and b. Then find sin(θ), cos(θ) and tan(θ), sin(φ), cos(φ) and tan(φ).



Solution:

(a) $h=\sqrt{a^2+b^2}$

(b) $\sin(\theta)=\frac{b}{h}$ $\cos(\theta)=\frac{a}{h}$ $\tan(\theta)=\frac{b}{a}$

(c) $\sin(\phi)=\frac{a}{h}$ $\cos(\phi)=\frac{b}{h}$ $\tan(\phi)=\frac{a}{b}$

(3) A vector $\vec{A}$ is given by $\vec{A}=5\hat{i}+4\hat{j}$. Find the following:
(a) what is the magnitude of $\vec{A}$?
(b) what is the angle the vector makes with the x-axis?
(c) what is the angle the vector makes with the y-axis?
(d) Express this vector using the "hat" notation.
(e) Express this vector using the "x-y" unit vector notation.

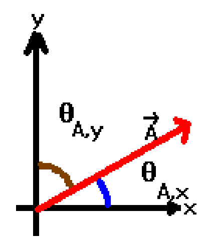
Solution:

(a) $|\vec{A}|=\sqrt{(\vec{A}\cdot\vec{A})}=[25+16]^{\frac{1}{2}}=\sqrt{41}=6.403$.

(b)
$$\cos(\theta_{A,x})=\frac{\vec{A}\cdot\hat{x}}{|\vec{A}|}=\frac{(5\hat{i}+4\hat{j})\cdot(\hat{x}+0\hat{y})}{\sqrt{5^2+4^2}}=\frac{5}{6.403}=0.7809\Rightarrow\theta_{A,x}=\cos^{-1}(0.7809)=38.66^0$$

(c)
$$\cos(\theta_{A,y})=\frac{\vec{A}\cdot\hat{y}}{|\vec{A}|}=\frac{(5\hat{i}+4\hat{j})\cdot(0\hat{x}+\hat{y})}{\sqrt{5^2+4^2}}=\frac{4}{6.403}=0.6247\Rightarrow\theta_{A,y}=\cos^{-1}(0.6247)=51.34^0$$

My unusual notation for these angles is for clarity: these are angles with respect to the particular axis of interest. Also, note that these 2 angles add up to be 90 degrees.



(d) $\vec{A}=5\hat{i}+4\hat{j}$

(e) $\vec{A}=5\hat{x}+4\hat{y}$

(4) Suppose a vector $\vec{B}$ is given by $\vec{B}=3\hat{i}+2\hat{j}$. Find the following:

(a)What is $2\vec{B}$?

(b) What is $\vec{B}+\vec{A}$?

(c) What is $\vec{B}-\vec{A}$?

(d) What is $\vec{B}\cdot\vec{A}$? (dot product)

(e) What is the angle made with respect to the positive x-axis by $\vec{B}+\vec{A}$?

Solution:

(a) $2\vec{B}=(2\times3)\hat{i}+(2\times2)\hat{j}=6\hat{i}+4\hat{j}$

(b) $\vec{B}+\vec{A}=[3\hat{i}+2\hat{j}]+[5\hat{i}+4\hat{j}]=(3+5)\hat{i}+(2+4)\hat{j}=8\hat{i}+6\hat{j}$

(c) $\vec{B}-\vec{A}=[3\hat{i}+2\hat{j}]-[5\hat{i}+4\hat{j}]=(3-5)\hat{i}+(2-4)\hat{j}=-2\hat{i}-2\hat{j}$

(d) $\vec{B}\cdot\vec{A}=[3\hat{i}+2\hat{j}]\cdot[5\hat{i}+4\hat{j}]=(3\times5)+(2\times4)=15+8=23$

(e) $\cos(\theta_{\vec{B}+\vec{A},\hat{x}})=\dfrac{\vec{B}+\vec{A}}{|\vec{B}+\vec{A}|}\cdot\hat{i}=\dfrac{[8\hat{i}+6\hat{j}]}{\sqrt{8^2+6^2}}\cdot\hat{i}=\dfrac{8}{\sqrt{100}}=\dfrac{8}{10}=0.8\Rightarrow\theta=36.87^0$

Note that the power of the method in (e) gives you a manner to obtain the angle with respect to any unit vector. You can define a unit vector in an arbitrary direction easily as:

$$\hat{B}=\frac{\vec{B}}{|\vec{B}|}\ .$$

For example, for this particular vector, we have:

$$\hat{B}=\frac{3\hat{i}+2\hat{j}}{\sqrt{3^2+2^2}}=\frac{3}{\sqrt{13}}\hat{i}+\frac{2}{\sqrt{13}}\hat{j}=0.832\hat{i}+0.555\hat{j}\ .$$

To find the angle between the vectors A and B, you would then take, for example:

$$\cos(\theta_{\vec{A},\vec{B}})=\frac{\vec{A}}{|\vec{A}|}\cdot\frac{\vec{B}}{|\vec{B}|}=\hat{A}\cdot\hat{B}=\frac{[5\hat{i}+4\hat{j}]}{\sqrt{5^2+4^2}}\cdot[0.832\hat{i}+.0555\hat{j}]=\frac{[5\hat{i}+4\hat{j}]}{\sqrt{41}}\cdot[0.832\hat{i}+0.555\hat{j}]$$
$$=\frac{4.16+2.22}{\sqrt{41}}=0.996\Rightarrow\theta=4.87^0$$

You could, of course, use the law of cosines which states:

$$\cos(\theta_{\vec{A},\vec{B}})=\frac{-|\vec{B}-\vec{A}|-|\vec{A}|^2-|\vec{B}|^2}{2|\vec{A}||\vec{B}|}=\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}=\hat{A}\cdot\hat{B}=\frac{\vec{A}\cdot\hat{B}}{|\vec{A}|}$$

where I have used the fact that the vector pointing from $\vec{A}$ to $\vec{B}$ is given by

$$\vec{B}-\vec{A}\ .$$

(5) Suppose a vector $\vec{C}$ is given by $\vec{C}=8\,\hat{i}-9\,\hat{j}$. A person walks along vector $\vec{A}$, then vector $\vec{B}$, followed by vector $\vec{C}$. At the end of this journey, what is the displacement (vector) and the distance from the origin. You may assume all units are in m here. $\vec{A}=5\,\hat{i}+4\,\hat{j}$ $\vec{B}=3\,\hat{i}+2\,\hat{j}$

Solution:

$$\vec{D}=\vec{A}+\vec{B}+\vec{C}=[5\,\hat{i}+4\,\hat{j}]+[3\,\hat{i}+2\,\hat{j}]+[8\,\hat{i}-9\,\hat{j}]=(5+3+8)\,\hat{i}+(4+2-9)\,\hat{j}=16\,\hat{i}+(-3)\,\hat{j}$$

$$|\vec{D}|=\sqrt{\vec{D}\cdot\vec{D}}=\sqrt{16^2+3^2}=\sqrt{256+9}=\sqrt{265}=16.279$$

You can easily verify that $\vec{A}+\vec{B}+\vec{C}-\vec{D}=\vec{0}$.

The vector symbol over 0 is necessary because the result of adding two vectors must produce a vector.