Finding the phase of a simple harmonic oscillator

$$x = A\cos(\omega t + \phi) : v = -\omega A\sin(\omega t + \phi) : a = -\omega^2 A\cos(\omega t + \phi)$$

Suppose at t=0, you know x and v. How do you determine the phase?

$$x_0 = A\cos(\phi) : v_0 = -\omega A\sin(\phi)$$

$$\Rightarrow \frac{v_0}{x_0} = -\omega \frac{\sin(\phi)}{\cos(\phi)} = -\omega\tan(\phi) \Rightarrow \phi = \arctan\left[\frac{-v_0}{\omega x_0}\right]$$

However since $\tan(\phi) = \tan(\pi + \phi)$, we will have two possible values for Φ.

The possibilities are these

assume $\frac{-\pi}{2} \le \phi_p \le \frac{\pi}{2}$

Then:

$$x_0 \ge 0 ; v_0 \ge 0 \Rightarrow \phi = \phi_p$$

$$x_0 \le 0 ; v_0 \ge 0 \Rightarrow \phi = \phi_p + \pi$$

$$x_0 \ge 0 ; v_0 \ge 0 \Rightarrow \phi = \phi_p$$

$$x_0 \le 0 ; v_0 \le 0 \Rightarrow \phi = \phi_p + \pi$$

For example; suppose

$$x_0 = 1 ; v_0 = 2 ; \omega = 3 \Rightarrow \phi_p = \arctan\left[\frac{-2}{3}\right] = -33^0 \Rightarrow \phi = -33^0$$

$$x_0 = -1 ; v_0 = 2 ; \omega = 3 \Rightarrow \phi_p = \arctan\left[\frac{2}{3}\right] = 33^0 : \phi = \phi_p + 180 = 213^0$$

$$x_0 = 1 ; v_0 = -2 ; \omega = 3 \Rightarrow \phi_p = \arctan\left[\frac{2}{3}\right] = 33^0 : \phi = \phi_p = 33^0$$

$$x_0 = -1 ; v_0 = -2 ; \omega = 3 \Rightarrow \phi_0 = \arctan\left[\frac{-2}{3}\right] = -33^0 : \phi = \phi_p + 180 = 147^0$$

You can check your results by checking the sin and cosine:

if $x_0 < 0$ is $\cos\phi < 0$? and $x_0 > 0$ is $\cos\phi > 0$?
$v_0 < 0$ is $-\sin\phi < 0$? and $v_0 > 0$ is $-\sin\phi > 0$?

Finally: suppose

$$x_0 = 0 \Rightarrow \phi = \frac{-\pi}{2}(v_0 > 0) \text{ or } \phi = \frac{\pi}{2_0} = 0(v_0 < 0) : v_0 = 0 \Rightarrow \phi = 0(x_0 < 0) \text{ or } \phi = -\pi(x_0 > 0)$$