Worksheet 5a:Physics 210: Additional Motion problems

(1) A helicopter is seen to move upward with a position given by: $y = y_0 + 4t - 3t^2$.

(a) What is the velocity of the helicopter at all times?
(b) What is the vector acceleration of the helicopter at all times?
(c) If $y_0=0$, at what times is the position equal to zero?
(d) At what time is the velocity zero?

(2) An airplane makes an emergency medical drop at the South Pole. The plane is moving with a velocity of 70 m/s at a distance of 40m above the South Pole.
(a) How long (in time) before the plane is above the South pole must the package be released in order to hit the South Pole exactly?
(b) Where was the plane at this time?
(c) What was the impact velocity vector of the package?

(3) A ball is thrown upward at an angle of 60^0 with an initial velocity of 30 m/s.
(a) How long is the ball in flight?
(b) How high does the ball go above the ground?
(c) How far in the x-direction does the ball travel?
(d) What is the impact velocity vector of the ball?

(4) Assume the speed of sound is 343 m/s. A hiker dislodges a rock on the side of a cliff and immediately screams to a hiker below to watch out. If the hiker below if 50 m below the rock, how long will it take between the time that the hiker hears the warning and the rock strikes the position (previous) of the hiker below?

(5) Let's find out how much time is saved at two speeds. Suppose you are going on a 100 mile trip (160934 m). Compare the time that it takes for the trip when driving at two speeds, v_1=55 mph and v_2=60 mph.

While we are talking about driving, what is twice as exciting as 25 m/s in terms of stopping distances? Assume the acceleration in each case is -7.62 m/s^2.

(1) A helicopter is seen to move upward with a position given by: $y = y_0 + 4t - 3t^2$.

(a) What is the velocity of the helicopter at all times?

By comparison to the standard free-fall equation we have:

$$y = y_0 + v_{0,y} t + \frac{1}{2} a_y t^2 \Rightarrow v_{0,y} = 4\frac{m}{s} : \frac{1}{2} a_y = -3 \Rightarrow a_y = -6\frac{m}{s^2}$$

(b) What is the vector acceleration of the helicopter at all times?

$$\vec{a} = 0\hat{x} - 6\hat{y}\frac{m}{s^2}$$

(c) If $y_0=0$, at what times is the position equal to zero?

$$y = y_0 + 4t - 3t^2 \Rightarrow 0 = 0 + 4t - 3t^2 = t(4 - 3t) \Rightarrow \begin{matrix} t = 0\,s \\ \text{or} \\ t = \frac{4}{3} = 1.333\,s \end{matrix}$$

(d) At what time is the velocity zero?

By direct comparison, since the acceleration in the y direction is -6 m/s^2, we have:

$$v_y = v_{0,y} + a_y t \Rightarrow 0 = 4 - 6t \Rightarrow t = \frac{4}{6} = 0.666\,s$$

(2) An airplane makes an emergency medical drop at the South Pole. The plane is moving with a velocity of 70 m/s at a distance of 40m above the South Pole.
(a) How long (in time) before the plane is above the South Pole must the package be released in order to hit the South Pole exactly?

It is absolutely going to be easiest to choose time = 0 when the package is released. So:

$$y=y_0+v_{0,y}t-\frac{1}{2}gt^2 \Rightarrow 0=40+0-\frac{1}{2}gt^2 \Rightarrow t=\pm\sqrt{\frac{2x40}{g}}=\pm 2.86\,s=+2.86\,s$$

The correct sign here is + since that is pretty much where everything started here. So all times after that must be in the future.

(b) Where was the plane at this time that the package was released?

The x equation of motion is $x=x_0+v_{0.x}t$. So using the time above, what we want to solve for here is actually x_0 and not x. Do note, however, the x coordinate of the package and the x coordinate of the plane are the same. Thus:

$$x=x_0+v_{0,x}t \Rightarrow 0=x_0+v_{0,x}t \Rightarrow x_0=-v_{0,x}t=-70\frac{m}{s}x2.86\,x=-200\,m$$

(c) What was the impact velocity vector of the package?
Using the 3rd equation, we have:

$$v_y^2=v_{0,y}^2-2g\Delta y \Rightarrow v_y^2=\pm\sqrt{-2x9.8x(0-40)}=-28\frac{m}{s}$$ (the negative sign is physical here)

We can also get this from the second equation:

$$v_y=v_{0,y}-gt=0-gt=-(9.8)x(2.86)=-28.02\frac{m}{s}$$

So the velocity vector is given by:

$$\vec{v}_f=v_{f,x}\hat{i}+v_{f,y}\hat{j}=[70\hat{x}-28\hat{y}]\frac{m}{s}$$

(3) A ball is thrown upward at an angle of 60^0 with an initial velocity of 30 m/s.
(a) How long is the ball in flight?
$v_0 = v_{0,y} - gt$. I am assuming that at t=0, the ball is thrown upward. At the end, the y component of the velocity is negative the initial component so

$$-v_{0.y} = y_{0,y} - gt \Rightarrow t = \frac{2v_{0,y}}{g} = \frac{2 x 30 x \sin(60^0)}{g} = +5.3 s$$

There are other ways to work this: at the top: $v_y=0$ so:

$$0 = v_{0,y} - gt_{1/2} \Rightarrow t_{1/2} = \frac{v_{0,y}}{g} \Rightarrow t = \frac{2v_{0,y}}{g} = \frac{2 x 30 x \sin(60^0)}{g} = +5.3 s$$

(b) How high does the ball go above the ground?
Using the 3rd equation we have:

$$v_y^2 = v_{y,0}^2 - 2g\Delta y \Rightarrow \Delta y = \frac{0^2 - [30\sin(60)]}{-2g} = 34.4 m$$

We could get this from the first equation ($t_{1/2}$= 2.65 s):

$$y = 0 + v_{0,y} t - \frac{1}{2} g t^2$$

(c) How far in the x-direction does the ball travel?

$$x = x_0 + v_{0,x} t = 0 + v_0 \cos(60^0) t = 79.5 m$$

(d) What is the impact velocity vector of the ball?

$$\vec{v}_f = v_{0,x}\hat{x} - v_{0,y}\hat{y} = [15\hat{x} - 26\hat{y}]\frac{m}{s}$$

(4) Assume the speed of sound is 343 m/s. A hiker dislodges a rock on the side of a cliff and immediately screams to a hiker below to watch out. If the hiker below if 50 m below the rock, how long will it take between the time that the hiker hears the warning and the rock strikes the position (previous) of the hiker below?

Let the hiker below be at y=0 m and the hiker above be at y=50 m. Then, the first equation of motion becomes for sound:

$$0=y_0+v_s t_s \Rightarrow t_s=\frac{50}{343}=0.14\,s$$

The equation for the rock becomes:

$$y=y_0+v_{0,r}t_r-\frac{1}{2}g t_r^2 \Rightarrow 0=50+0-\frac{1}{2}g t_r^2 \Rightarrow t_r=\pm\sqrt{\frac{2\,x\,50}{g}}=3.19\,s$$

So the time the hiker below has to react is:

$$T_{react}=t_r-t_s=3.19-0.14=3.05\,s$$

This does not account for the reaction time of about 0.2 s.

(5) Let's find out how much time is saved at two speeds. Suppose you are going on a 100 mile trip (160934 m). Compare the time that it takes for the trip when driving at two speeds, v_1=55 mph and v_2=60 mph.

The equation of motion is given for the two speeds. In both cases, the distance traveled is the same.

$$x_1=v_1 t_1 : x_2=v_2 t_2 : x_1=x_2\equiv x$$

Solve these for the times:

$$t_1=\frac{x}{v_1} : t_2=\frac{x}{v_2}$$

The difference in times is then:

$$\Delta t=|t_2-t_1|=x\left\|\left[\frac{1}{v_2}-\frac{1}{v_1}\right]\right\|$$

As an example, let v_1 be 55 mph (24.6 m/s) and v_2 is 70 mph (31.3 m/s). Then:

$$\Delta t=100\left\|\left(\frac{1}{70}-\frac{1}{55}\right)\right\|=0.39\,hr=23\,min$$

How about the same trip at 55 mph and 60 mph?

$$\Delta t=0.15\,hr=9.1\,min$$

While we are talking about driving, what is twice as exciting as 25 m/s in terms of stopping distances? Assume the acceleration in each case is -7.62 m/s^2.

In terms of stopping distances, assume the accelerations are the same for both cars, but one is at 25 m/s and the other is traveling at such a speed so that the stopping distance is twice that of the first car.
The equations of motion are:

$$0=v_1^2-2a\Delta x_1 : 0=v_2-2a\Delta x_2 : \Delta x_2=2\Delta x_1$$

$$\Rightarrow v_1^2=2a\Delta x_1 : v_2^2=2x2a\Delta x_2$$

$$\Rightarrow\left(\frac{v_1}{v_2}\right)^2=\frac{2a\Delta x_1}{2x2a\Delta x_1}=\frac{1}{2}\Rightarrow\frac{v_1}{v_2}=\frac{1}{\sqrt{2}}\Rightarrow v_2=\sqrt{2}v_1$$

So to double the stopping distance, you do not double the speed. It is only $\sqrt{2}$ higher. Let's find a typical stopping distance.

Suppose the speed initially was 25 m/s (56 miles/hr) and the acceleration was 7.62 m/s^2 (23 ft/s^2) . Then the stopping distance is given by: $\Delta x_1=\frac{v_1^2}{2a}=\frac{25^2}{2x7.62}=41\,m$. The speed at which the stopping distance doubles is given by: $v_2=25\sqrt{2}=35.3\,m/s(\text{ or }79.2\,mi/hr)$. So the conclusion is this: while you might think 110 miles/hr is twice as exciting as 55 miles per hour, in fact it is 4 times more exciting. 80 mi/hr is about twice as exciting.