| r2020 | Physics 210: Worksheet r10 | Name __________________ |
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(1) Suppose a mass m slides down a frictionless inclined plane (of angle $\theta$). At the bottom of the plane, the mass encounters a flat surface with a coefficient of friction $\mu$. How far does the mass move beyond the bottom of the plane if it falls through a vertical height y? Provide numerical answers for the case y=1m and $\mu$=0.3.

(2) A cyclist approaches the bottom of a hill at a speed of 11 m/s. The hill is 6 m high. Ignoring friction, how fast is the cyclist moving at the top of the hill? What is the meaning of the general solution if the hill is 9 m high? :)

(3) **Using energy considerations**, what is the speed of a rock when it has fallen through a distance of 100 m if it started from rest?

(4) Suppose a 100 kg mass is traveling with a velocity of 15 m/s. If the mass strikes a spring with a spring constant k=5 N/m, how much will the spring compress before the mass stops. This system is horizontal.

(5) Suppose a 100 kg mass is traveling with a velocity of 15 m/s. If the mass strikes a spring with a spring constant k=5 N/m, how much will the spring compress before the mass stops. This system is horizontal and the spring is initially compressed 30 m.

(1) Suppose a mass m slides down a frictionless inclined plane (of angle $\theta$). At the bottom of the plane, the mass encounters a flat surface with a coefficient of friction $\mu$. How far does the mass move beyond the bottom of the plane if it falls through a vertical height y? Provide numerical answers for the case y=1m and $\mu$=0.3.

You want to apply energy conservation here. The equation is then:

$$\Delta K_{NC} = \Delta K_C + \Delta U$$

Now, looking at the problem from beginning to end, you see that in fact we do have the case: $DK = 0$

We also have: (and this is only along the flat surface):

$$\Delta K_{NC} = W_f = -\mu m g (\Delta x)$$

We also for the gravitational potential energy have:

$$\Delta U = -mgy$$

So let's put it all together:

$$\Delta K_{NC} = \Delta K_C + \Delta U \Rightarrow -\mu m g \Delta x = -mgy \Rightarrow \Delta x = \frac{y}{\mu}; \Delta x = \frac{1}{0.33} = 3.03\,m$$

(2) A cyclist approaches the bottom of a hill at a speed of 11 m/s. The hill is 6 m high. Ignoring friction, how fast is the cyclist moving at the top of the hill?

Solution: Total mechanical energy is conserved here. Thus, $\Delta U + \Delta K_C = 0$ . This problem is a bit unlike other problems since the kinetic energy is not zero at any time in this problem. We need to calculate each of the relevant terms here. The terms are:

$$\Delta U = mgy - 0 = mgy$$

and

$$\Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 \quad .$$

Let's now write the energy conservation equation:

$$\Delta K_C + \Delta U = \Delta K_{NC} \Rightarrow \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 + mgy = 0$$

As a note: there is certainly static friction involved here. However there is no displacement of the wheel with the road so there is no work done against static friction.

No you want to solve this for the final velocity. Thus:

$$\frac{1}{2} m v_f^2 = \frac{1}{2} m v_0^2 - mgy \Rightarrow v_f^2 = v_0^2 = 2gy \Rightarrow v_f = \pm \sqrt{v_0^2 - 2gy}$$

The solution for the final velocity is then:

$$v_f = \pm \sqrt{v_0^2 - 2gy} \quad .$$

For this particular numerical example, we then get:

$$v_f = + \sqrt{v_0^2 - 2gy} = \sqrt{121 - 117.6} = \sqrt{3.4} = 1.84\,m/s \quad .$$

We are only asked for "how fast" in this problem so we choose the positive sign.
You might ask yourself: what happens if instead of 6 m you were 7 m up the hill. Then,

$$v_f = + \sqrt{v_0^2 - 2gy} = \sqrt{121 - 137.2} = \sqrt{-16.2} = 4\,i\,m/s$$

The "I" is imaginary and it means that the answer is not physical. In fact, the highest point at which the problem remains physical is when

$$v_0^2 - 2gy = 0 \Rightarrow y_{max} = \frac{v_0^2}{2g} \quad .$$

(3) Using energy considerations, what is the speed of a rock when it has fallen through a distance of 100 m if it started from rest?
Solution: Total mechanical energy is conserved here. Thus,

$$\Delta U+\Delta K_C=\Delta K_{NC} \quad .$$

We need to calculate each of the relevant terms here:

$$\Delta U=-mgy-0=-mgy$$

$$\Delta K_{NC}=0$$

and

$$\Delta K_C=\frac{1}{2}mv^2-0=\frac{1}{2}mv^2 \quad .$$

Now put everything together:

$$-mgy+\frac{1}{2}mv^2=0\Rightarrow v=\pm\sqrt{2gy}$$

The – solution is the physical solution here. Putting in the values, we find:

$$v=-\sqrt{2x9.8x100}=-\sqrt{1960}=-14.7=-44.3\frac{m}{s} \quad .$$

(4) Suppose a 100 kg mass is traveling with a velocity of 15 m/s. If the mass strikes a spring with a spring constant k=5 N/m, how much will the spring compress before the mass stops. This system is horizontal.
This problem conserves total mechanical energy. Thus:

$$\Delta U + \Delta K_C = \Delta K_{NC} \quad .$$

We need to calculate each of the relevant terms.

$$\Delta K_{NC} = 0$$

$$\Delta K_C = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = 0 - \frac{1}{2} m v_i^2 = \frac{-1}{2} m v_i^2 \quad .$$

On, and now the one you need to be real careful with:

$$\Delta U = \Delta U_s = \frac{1}{2} K (\Delta x)^2 = \frac{1}{2} k (x_f - x_i)^2$$

And I really strongly suggest, even if you have to work the problem several times, do so and keep the initial position of the spring (or the final position) equal to zero. However if you can not do this, the problem is going to get harder to solve and you need to be really careful when you solve a quadratic equation. In particular, dropping a ball on an uncompressed spring, or the next one; that is going to be a little bit harder to solve.

Now put everything together:

$$0 = -\frac{1}{2} m v_i^2 + \frac{1}{2} k x_f^2$$

Solve this for the final position of the spring:

$$\frac{1}{2} m v_i^2 = \frac{1}{2} k x_f^2 \Rightarrow x_f = \pm \sqrt{\frac{m}{k}} v_i$$

The final position of the mass is then:

$$x_f = \pm \sqrt{\frac{m}{k}} v_i \quad .$$

But in this problem, we’re asked “how much will the spring compress” so we’re looking for $\Delta x$. The symbolic problem solution is then:

$$\Delta x = \pm \sqrt{\frac{m}{k}} v_i$$

If the mass is initially traveling in the +x direction, then the physical solution is the positive. I’ll assume the mass is moving in the +x direction initially. Then the physical solution is +.
Let’s put in the values to find then

$$\Delta x = \sqrt{\frac{100\,kg}{5\,N/m}} 15 \frac{m}{s} = 15\sqrt{20} = 67.1\,m$$

(5) Suppose a 100 kg mass is traveling with a velocity of 15 m/s. If the mass strikes a spring with a spring constant k=5 N/m, how much will the spring compress before the mass stops. This system is horizontal and the spring is initially compressed 30 m.

The spring was already compressed through a distance or 30 m and the mass causes it to compress an additional x m.

The final potential energy of the spring was:

$$U_f=\frac{1}{2}k(x+30)^2$$

The initial potential energy of the spring was (before it was struck):

$$U_i=\frac{1}{2}k(30)^2 \quad .$$

The change in the potential energy was:

$$\Delta U_s=U_f-U_i=\frac{1}{2}k(x+30)^2-\frac{1}{2}k(30)^2=\frac{1}{2}k(x^2+60x+900-900)=\frac{1}{2}k(x^2+60x)$$

All of this change comes from the change in kinetic energy of the mass:

$$\Delta K_C=K_f-K_i=0-\frac{1}{2}mv_i^2=-\frac{1}{2}mv_i^2=\frac{-1}{2}x100x(15)^2=-11250\,J$$

So we actually do need to solve the equation:

$$\frac{1}{2}kx^2+30kx-11250=0$$

$$x^2+60x-\frac{22500}{k}=0$$

$$\Rightarrow x^2+60-\frac{22500}{5}=x^2+60-4500=0$$

$$x=\frac{-60\pm\sqrt{60^2+4x4500}}{2}$$

$$x=-30\pm\frac{1}{2}\sqrt{3600+18000}$$

$$=-30\pm\frac{1}{2}\sqrt{21600}=-30\pm\frac{147}{2}=-30\pm73.5=-103.5 \text{ or } 43.5$$

If we choose the negative solution, this would mean that the spring acted in a non-physical manner. If we choose the positive solution then X=43.5 m. The total compression of the spring would be 73.5m. All in all, you really need to be careful about superimposition of energies, however. In particular, from classical physics relative velocities (in one dimension) will add but the result is that the kinetic energy is not what is given by K1+K2. For example: if a 1kg mass is traveling with a speed of 2 m/s and a 1 kg mass is traveling at a speed of 4 m/s, the total kinetic energy of the second mass is not 4 J; instead it is 8 J.