r20	Physics 210: Worksheet 03	Name _________________

Each of these problems concerns the case where acceleration is a constant.

Assume $\vec{a} = -|g|\hat{y} = -|g|\hat{j} = -9.8\,\hat{y}\,\frac{m}{s^2}$

(1) From a Lyon College dorm room, a student[1] drops a water balloon which hits the ground 10 m below. How long was the balloon in the air if it was released from rest?

[1] Not one of my students.

(2) A ball which is thrown straight up is observed to reach 20 m at its maximum height. What was the height where the ball had one-third of its initial speed?

(3) A ball is dropped from rest through a distance of 20 m. How fast is the ball moving when it hits the ground?

(4) Suppose the ball in (3) was given a velocity in the x direction of 5 m/s. How far in the x direction will the ball travel before it hits the ground?

(5) A ball is thrown upward from the ground at an angle of 15^0 with an initial velocity of 30 m/s. (a) How high will the ball travel (b) how far in the x direction will the ball travel (c) what will be the impact velocity vector and (d) how long will the ball be in the air?

(1) From a Lyon College dorm room, a student[1] drops a water balloon which hits the ground 10 m below. How long was the balloon in the air if it was released from rest?

[1] Not one of my students.

Solution: $y=y_i+v_{y,i}t+\frac{1}{2}a_y t^2$. Here, $v_{y,i}=0$ and (this is important) $\vec{a}=-g\hat{j}\Rightarrow a_y=-g$ where g=+9.8 m/s^2. Also, y_i=+10 m and y=0 when the balloon hits the ground. Thus our equation of motion becomes: $0=+10+0-\frac{1}{2}gt^2\Rightarrow -10=\frac{-1}{2}gt^2$. Let's solve this for t: $gt^2=20\Rightarrow t=\pm\sqrt{\frac{20}{g}}=\pm\sqrt{\frac{20}{9.8}}=\pm 1.429\,s$. Which sign is correct here is the + sign.

(2) A ball which is thrown straight up is observed to reach 20 m at its maximum height. What was the height where the ball had one-third of its initial speed?

Solution: Note at the maximum height, the velocity in the y direction is zero. Then it is relatively easy to find the initial velocity from the third equation:

$$v_{y,f}^2=v_{y,i}^2+2a(\Delta y)=v_{y,i}^2-2g(\Delta y)$$

$$\Rightarrow 0=v_{y,i}^2-2g(\Delta y)\Rightarrow v_{y,i}=\pm\sqrt{2g(\Delta y)}=\pm\sqrt{2\,x\,9.8\,x\,20}=\pm\sqrt{392}=\pm 19.8\frac{m}{s}$$

The + sign is correct here since the ball is thrown upward. One third of this initial speed is given by:

$$v_{y,\frac{1}{3}v_{y,i}}=\frac{1}{3}v_{y,i}=\frac{1}{3}(19.80)=6.6\frac{m}{s}$$

So the height at this point is given by:

$$v_{y,\frac{1}{3}v_{y,i}}^2=v_{y,i}^2+2a(\Delta y_{1/3})=v_{y,i}^2-2g(\Delta y_{1/3})\Rightarrow \Delta y_{1/3}=\frac{-6.6^2-19.8^2}{2(9.8)}=17.8\,m$$

(3) A ball is dropped from rest through a distance of 20 m. How fast is the ball moving when it hits the ground?

Solution: $v_y^2=v_{y,i}^2+2a(\Delta y)$. Here, we have freefall so $\vec{a}=-g\hat{j}\Rightarrow a_y=-g$ where $g=+9.8\frac{m}{s^2}$ and $v_{y,i}=0$. We also have that $\Delta y=y_f-y_i=0-20=-20\,m$. Thus, using these values in the equation of motion:

$$v_y^2=v_{y,i}^2+2a(\Delta y)\Rightarrow v_y^2=-2g(-20)=40g=392\frac{m^2}{s^2}\Rightarrow v=\pm 19.8\frac{m}{s}$$.

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Compare this problem to problem 2! Note that the velocity vector is given by $\vec{v}=0\hat{i}-19.8\hat{j}$ so the - sign is actually the correct sign here! Never-the-less, notice the symmetry here.

(4) Suppose the ball in (3) was given a speed in the x direction of $5\frac{m}{s}$. How far in the x direction will the ball travel before it hits the ground?

Solution: You need to find out how long the ball was actually falling, then this time will be used to get the displacement in the x direction. The time comes, easily enough from the following: $x=v_x t=5.0\frac{m}{s}(2.02\,s)=10.1\,m$. Here, we have freefall so $\vec{a}=-g\hat{j}\Rightarrow a_y=-g$ where $9=9.8\,m/s^2$. $v_{y,i}=0$ and $30\,m/s$ from problem 3. Thus, solve for t: $v_y=v_{y,i}+a_y t\Rightarrow -19.8=-g t\Rightarrow t=\frac{19.8}{9.8}=2.02\,s$. The distance in the x-direction is then given by $x=v_x t=5.0\frac{m}{s}(2.02\,s)=10.1\,m$.

(5) A ball is thrown upward from the ground at an angle of 15^0 with an initial speed of $30 m/s$. (a) How high will the ball travel (b) how far in the x direction will the ball travel (c) what will be the impact velocity **vector** and (d) how long will the ball be in the air?

Solution: this problem puts together many things. Let's look at the symmetries involved: $v_{y,i}=-v_{y,f}$ so the angle of launch is the same as the angle of impact. Also, the speed of launch is the same as the speed at impact. One other symmetry is that the time for the ball to reach the maximum is the same as the time for it to come back down. Finally, don't forget that the y-component of the velocity is zero at the highest point.

Now, to solve this problem, first find the y-component of the initial velocity: and $v_x=v_{x,i}=|\vec{v}_{t=0}|\cos(\theta)$. The reason that is that there is no acceleration in the x direction. We can easily determine these to be: $v_{y,i}=7.76\frac{m}{s}$ and $v_{x,i}=v_x=29.0\frac{m}{s}$. Now that we have the initial velocity, we can answer the various parts:

(a) Use $v_y^2=v_{y,i}^2+2a_y(\Delta y)$. At the top, $v_y=0$. Since we are discussing freefall, $\vec{a}=-g\hat{j}\Rightarrow a_y=-g$ where $g=+9.8\frac{m}{s^2}$, and $\Delta y=y_f-y_i=y_f$ since $y_i=0$ (I am setting zero to be at the ground here). Thus, $0=(7.76)^2-2gy_f\Rightarrow y_f=\frac{60.22}{19.6}=3.07m$.

(b) In order to answer (b) it is necessary to answer (d) first.

(d) This time could be found from $v_y=v_{y,i}+a_y t$. Since we are discussing freefall, $\vec{a}=-g\hat{j}\Rightarrow a_y=-g$ where $g=+9.8\frac{m}{s^2}$, and at the top, $v_y=0$. This gives the answer as:

$$0=v_{y,i}-gt\Rightarrow t=\frac{v_{y,i}}{t}=\frac{7.76}{9.8}=0.792s$$

$0=v_{yi}-gt\Rightarrow t=\frac{v_{yi}}{g}=\frac{7.76}{9.8}=0.792s$. This is only (1/2) of the total time. the total time is: $t_{total}=2x0.792=1.58s$.

Notice that this time could also be obtained by solving for the times at which the y position were equal to zero to give the same result:

$$y=y_i+v_{y,i}t+\frac{1}{2}a_y t^2\Rightarrow t=0\vee t=\frac{-2y_{y,i}}{a_y}.$$

Back to (b) we can find the distance from $x=x_i+v_{x,i}t+\frac{1}{2}a_x t^2$. There is no acceleration in the x direction so $a_x=0$. Also, x_i is zero. Thus, $x=v_{x,i}t=29x1.58=45.8m$.

(c) The impact velocity vector will be $\vec{v}_f=29.0\hat{i}-7.76\hat{j}\frac{m}{s}$ (by looking at the symmetry of the problem).