



$x=0, y=0, x=a V=0. y=b V=V_0$

Solution:

Subtract the constant V_1 from all potentials. then V_0-V_1 is the potential at b . Rewrite as the usual solution, at then end, superimpose the potentials.

$$V(x, y) = (A \sin(kx) + B \cos(kx))(C e^{ky} + D e^{-ky})$$

$$@ x=0, V=0 \Rightarrow B=0$$

$$@ x=a, V=0 \Rightarrow ka = n\pi \Rightarrow k = \frac{n\pi}{a}$$

$$\text{at } y=0, V=0 \Rightarrow C = -D$$

$$V(x, y) = \sum_{n=0}^{\infty} A_n \sin\left(n\pi \frac{x}{a}\right) \sinh\left(n\pi \frac{y}{a}\right)$$

$$\int_{x=0}^{x=a} V(x, y) \sin\left(m\pi \frac{x}{a}\right) dx = \sum_{n=0}^{\infty} A_n \sinh\left(n\pi \frac{y}{a}\right) \int_{x=0}^{x=a} \sin\left(n\pi \frac{x}{a}\right) \sin\left(m\pi \frac{x}{a}\right) dx = A_m \sinh\left(m\pi \frac{y}{a}\right) \frac{a}{2}$$

$$\int_{x=0}^a \sin\left(m\pi\frac{x}{a}\right) dx = -\left[\frac{a}{m\pi} \cos\left(m\pi\frac{x}{a}\right)\right]_0^a = \frac{-a}{m\pi} [(-1)^m - 1] = \frac{2a}{m\pi} \begin{cases} 1; m \text{ odd} \\ 0; m \text{ even} \end{cases}$$

$$\frac{V_0}{\sinh\left(m\pi\frac{b}{a}\right)} \frac{2a}{m\pi} = A_m \quad (m \text{ odd})$$

$$V(x, y) = \sum_{n=1,3,5,\dots}^{\infty} \frac{2aV_0}{n\pi} \frac{\sinh\left(n\pi\frac{y}{a}\right)}{\sinh\left(n\pi\frac{b}{a}\right)} \sin\left(n\pi\frac{x}{a}\right)$$

Now superimpose the potentials

$$V(x, y) = V_1 + \sum_{n=1,3,5,\dots}^{\infty} \frac{2a(V_0 - V_1)}{n\pi} \frac{\sinh\left(n\pi\frac{y}{a}\right)}{\sinh\left(n\pi\frac{b}{a}\right)} \sin\left(n\pi\frac{x}{a}\right)$$