1.3.2 – 1.3.6 Fundamental Theorems for a function.

1.3.2: Fundamental theorem of calculus.

$$\int_a^b \frac{df}{dx} dx = f(b) - f(a)$$

Which means to integrate, you create a function whose derivative is f.

1.3.3: Fundamental theorem for gradients.

$$\oint_a^b \vec{\nabla} T \cdot d\vec{L} = T(b) - T(a)$$

The line integral of a derivative of a function is the value of the function at the boundaries.

It is very important to note that if the stuff inside the line integral can be written as the gradient of some function, then (1) the result is independent of path and (2) the result if it is a closed path is zero. This implies that T is a conservative.

Do not forget that $d\vec{L} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$.

1.3.4 Fundamental Theorem for Divergences

This goes by lots of names: Gauss's theorem, Green's theorem of the diveregence theorem.

$$\oiiint_{volume} \vec{\nabla} \cdot \vec{v}\, d^3 r = \oiint_{surface} \vec{v} \cdot d\vec{A}$$

Again, this says that the integral of a derivative (the divergence) is equal to the value of the function at the boundary. That is why the surface has to bound the volume and the volume is a closed surface.

1.3.5 Fundamental Theorem for Curls

$$\oiint_{surface} (\vec{\nabla} x \vec{v}) \cdot d\vec{A} = \oint_{path} \vec{v} \cdot d\vec{L}$$

This is Stoke's theorem and it says that the integral of a derivative over a region is equal to the value of the function at the boundary. Specifically this is a closed path that surrounds the surface. If you do integrate over the curl of a vector function such as this, then (1) the integral depends only on the boundary (emphasizing NOT on the particular surface) and (2) for any closed surface the value of the integral is zero, which means: $\oiint_{closed\ surface} (\vec{\nabla} x \vec{v}) \cdot d\vec{A} = 0$. Your author talks about the mouth of a balloon closing to a point which I think is a good analogy. This would necessarily involve a 3-d type of surface. But do note not all surfaces are closed such as in example 1.11.

1.3.6 Integration by Parts

This is extremely useful when you need it.

The product rule states:

$$\frac{d}{dx}(fg)=f\left(\frac{dg}{dx}\right)+g\left(\frac{df}{dx}\right)$$

Now we want to integrate both sides:

$$\int_a^b \frac{d}{dx}(fg)\,dx=fg|_a^b=\int_a^b f\left(\frac{dg}{dx}\right)dx+\int_a^b g\left(\frac{df}{dx}\right)dx \quad .$$

So, rearranging this gives:

$$\int_a^b f\left(\frac{dg}{dx}\right)dx=-\int_a^b g\left(\frac{df}{dx}\right)dx+fg|_a^b$$