

Standing transverse harmonic waves on a string New: Fall 2020 Pandemic version

This lab is a bit of a departure from the normal type of lab that we have done this semester. A significant part of this lab is the ability to recognize standing waves.

We will see in class that the wave form of harmonic traveling waves on a string are given by:

$$f(x,t) = A \cos(x \pm vt) .$$

I will show in class that the “-” sign corresponds to a wave traveling in the + x direction while the “+” sign corresponds to a wave traveling in the -x direction.

Non-dispersive harmonic waves are characterized by

$$f \lambda = v .$$

where f is the frequency in Hz, λ is the wave length and v is the wave speed.

I want to show you how this comes about. Consider the rate equation:

$$\text{distance} = \text{rate} \times \text{time} .$$

The time we are going to consider is the period which is the amount of time required for a wave to exactly duplicate a previous state. It of course has units of [s]. The rate is in the context of harmonic traveling waves call the wave speed and has typical SI units of [m/s]. The distance the wave travels in the time of one period is the wavelength (λ) of the wave. In terms of the symbols λ (wavelength) , T (period), and v (wavespeed) we have then:

$$\lambda = v T$$

When talking about waves, it is more appropriate to talk in terms of frequency (f):

$$\lambda = v T \Rightarrow \frac{\lambda}{T} = v : f = \frac{1}{T} \Rightarrow f \lambda = v$$

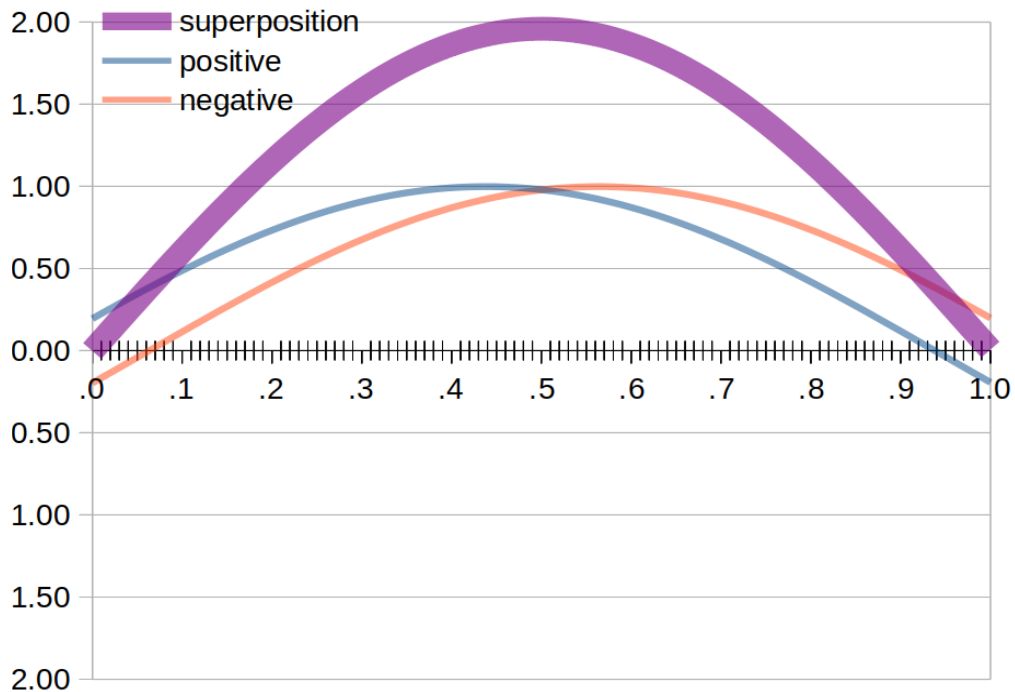
We will also see in class that for transverse oscillations on a string, the wave speed is given by:

$$v = \sqrt{\frac{\text{tension}}{\text{mass per unit length}}} = \sqrt{\frac{F}{\mu}}$$

where F is the tension [N] and μ is the mass per unit length [kg/m].

Standing waves on a string

Between two nodes, the lowest mode of oscillation is shown below.



The nodes are the fixed points that show no motion. The area of greatest displacement is the amplitude. This shows only $\frac{1}{2}$ of a wave length. The scale is a length of 1 meter so the wave length here is 2 m. When a wave strikes a rigid boundary, it reflects and travels in the opposite direction with a 180° phase reversal. This is the only type of boundary condition that we will consider here. The blue wave is traveling towards the right and the red wave is the reflected wave traveling towards the left. The thick purple wave is given by the superposition (including the 180° phase shift upon reflection). In order to produce a standing wave, there needs to be a node at each end. As a note: this is the case for a string with rigid boundary conditions on each end.

note that I have placed hints and helps in the "exp1, exp2 and exp3" cells.
Also make sure you always copy and paste special number!

Experiment 1

In experiment 1, you will keep the wavelength fixed at 2 m. Usually in the lab, we can not vary the frequency because the equipment required is very expensive (you would need a programmable current source). However for the simulation, you can vary the frequency. So the range of things we can look at is more extensive. You will pick a (pick a fairly small one) mass per unit length. You will then change the frequency from 10 to 60 Hz, and vary the tension to make standing waves. Here is the theory for this portion of the lab:

$$\text{Since } v = \sqrt{\frac{F}{\mu}} \Rightarrow f\lambda = \sqrt{\frac{F}{\mu}} \Rightarrow f^2\lambda^2 = \frac{F}{\mu} \Rightarrow f^2 = \left[\frac{1}{\lambda^2\mu} \right] F$$

$$\text{Let } y \equiv f^2$$

A plot of

$$y = \left[\frac{1}{\lambda^2\mu} \right] F$$

Should give a straight line. Since the wavelength is set to be 2 meters (you will need to make sure you produce a standing wave at the lowest mode for this to happen) we can make a measurement of μ . This is then compared to the theoretical value that you chose at the beginning of the experiment by % error. I also show the R^2 on the graphical analysis for this.

Experiment 2

In experiment 2, you are going to choose a mass per unit length. I recommend somewhere around .01 but do not choose my numbers. You will also choose a tension (around 50 but not my numbers). The idea here is start at the lowest frequency which gives a wavelength of 2 m. Increase the frequency getting shorter and shorter wavelengths on the string. You may need to zoom in on the scale in order to measure the wave length. Also be sure you have a node again at each end so that you have a true standing wave.

Here is how the analysis goes for this: for transverse oscillations on a string, we have again:

$$v = \sqrt{\frac{F}{\mu}} \Rightarrow f\lambda = \sqrt{\frac{F}{\mu}} \Rightarrow f^2\lambda^2 = \frac{F}{\mu} \Rightarrow f^2 = \left[\frac{1}{\lambda^2\mu} \right] F$$

Ok, so far I have just pasted the earlier equation. Now, we are going to keep the tension and the mass per unit length constant, and thus the wave speed will be constant. So we in fact have:

$$f\lambda = v \Rightarrow \lambda = \frac{v}{f} = v(\text{period})$$

so if a plot of λ vs $1/f$ (which is period) is made, we would expect to see a straight line and the slope of this would be the wave speed. From your chosen mass per unit length and tension, the wave speed is then calculated and compared to the measured value by %error.

After you generate your frequencies that produce standing waves, I want you to then find a relationship between the lowest frequency of oscillation and the higher frequencies. Work on this until you see it because the relationship is quite simple and also quite important here and later.

Experiment 3

In experiment 3, you will keep the tension fixed and you will also keep a standing wavelength of 2 m. Do not choose my number for the tension there. We usually can not make these types of measurements in the lab but we sure can in a simulation because what you can do in experiment 3 is to change the mass per unit length of the string and find the frequency for standing waves. Practically how this could happen is by doubling the string, and twisting it, and so on. I suggest you start with a fairly small mass per unit length here and increment it by 0.001 kg/m. Find the frequencies required to keep the wavelength of standing waves at 2 m.

The analysis of this is as follows:

The wave speed is given by:

$$v = \sqrt{\frac{F}{\mu}} .$$

Then the frequency to produce a wave length of 2 meter is given by:

$$\lambda f = v \Rightarrow f = \frac{v}{\lambda} = \frac{1}{\lambda} \sqrt{\frac{F}{\mu}} \Rightarrow f^2 = \frac{F}{\lambda^2} \left(\frac{1}{\mu} \right)$$

Again as before, let

$$x \equiv \frac{1}{\mu} \quad \text{and let} \quad y = f^2$$

Then a plot of x vs y should provide a slope of

$$\text{slope} = \frac{F}{\lambda^2} \Rightarrow F = \lambda^2 \text{slope}$$

So there will be a measured tension and this will be compared to the tension which was originally chosen by means of %error.