

### Sample calculations for lab 07

Here, I want you to study how the results that give the period are obtained.

A mass  $m$  is attached to the end of a string of length  $L$  forming a simple pendulum. Initially the mass is at an angle  $\theta_0$ .

#### Period of the simple pendulum

The torque on the pendulum is initially given by (assuming the end of the string to be the pivot point:

I: Torque about the axis:  $\vec{\Gamma} = \vec{R} \times \vec{F} \Rightarrow |(\vec{\gamma})| = -|\vec{R}||\vec{F}|\sin(\theta)$

II: small angle approximation: if  $\theta$  is small, then  $\sin(\theta) \approx \theta$

III: response of system to the torque: it produces a time rate of change in angular momentum or:  $\Gamma = I\alpha$  where  $I$  is the moment of inertia. The moment of inertia about the pivot is  $I = mL^2$ .

From this find  $\alpha$  in terms of the torque.  $mL^2\alpha = -Lmg\sin(\theta) \Rightarrow \alpha \approx -\frac{g}{L}\theta$

IV: Equation of motion: Write the resulting equation in "standard form" as

$$\alpha + \frac{g}{L}\theta = 0$$

Recall that the general solution to this type of equation is:  $\theta = \theta_0 \cos(\omega t)$ ;  $\omega = \sqrt{\frac{g}{L}}$  when the pendulum is at an amplitude.

V: From  $\omega$ , find the period,  $T$ , of the simple pendulum when  $L = 1$  m.

$T =$  \_\_\_\_\_

#### Period of the Spring mass system

A mass  $m$  is connected to a spring of spring constant  $k$ . The system is held horizontal in the Earth's gravitational field.

I: the spring exerts a force on the mass given by:  $F = -kx$  (ignoring the - sign here).

II: the mass responds to this force by Newton's laws:  $F = ma$

III: Equate these to obtain the equation of motion in standard form:

$$ma = -kx \Rightarrow a + \frac{k}{m}x = 0$$

IV: Recognize the solution to this is:  $x = A \cos(\omega t)$ ;  $\omega = \sqrt{\frac{k}{m}}$  when the mass is initially at an amplitude.

V: From this find the period,  $T$ , of the spring mass system when  $k = 1$  N/m and  $m = .5$  kg.

$T =$  \_\_\_\_\_