Archimedes' Principle and pressure
Revised for Fall 2016

Archimedes' principle states that the buoyant force is equal to the weight of the fluid displaced. In equation form, we write this as:

$$F_b = W_{of\,fluid\,displaced} \quad .$$

For an object which displaces a volume V, the weight of the fluid displaced is given by:

$$W = \rho_{fluid} V g \quad ,$$

where ρ_{fluid} is the density of the fluid that the object is placed in. In the case of water, $\rho_{water} = 1x10^3 \frac{kg}{m^3}$. V is the volume **of the displaced fluid** and g is the acceleration due to gravity. If the object is completely submerged in the fluid, then

$$V = V_{object} \quad .$$

Let's examine the two obvious cases for an object (such as a metal cylinder) completely immersed in water and then for a block of wood floating on water. In the case of the cylinder, we find that the buoyant force is given by

$$F_b = \rho_{water} g V_{object} \quad .$$

Now, what will happen for the metal cylinder is that we would see the apparent weight decrease by the buoyant force or

$$\Delta Weight = g V_{cyliner} (\rho_{cyliner} - \rho_{water}) \quad .$$

In the case of the wooden block, since the block floats on water, we know immediately that the density of the wooden block is less than that of water. We can answer a different question then for the wooden block, namely how much of the wooden block would be immersed. This fraction, (which is a percentage when multiplied by 100) is obtained by equating the buoyant force to the weight of the block, or

$$F_b = W_{object} \quad .$$

Again, this is true for a floating object.

We can rewrite this in terms of volumes and densities as

$$F_b = \rho_{fluid} g V_{fluid\ displaced} = \rho_{fluid} g V_{object\ immersed}$$

where

$$W_{object} = \rho_{object} g V_{object} \quad .$$

We solve this for the fraction of the object immersed, we find that this is given by:

$$\text{Floats} \Rightarrow F_b = W_{oject}$$

$$\Rightarrow \rho_{fluid} g V_{fluid\ displaced} = \rho_{oject} g V_{object}$$

$$\Rightarrow \rho_{fluid} V_{fluid\ displaced} = \rho_{object} V_{object}$$

$$\Rightarrow \frac{V_{object\ immersed}}{V_{object}} = \frac{\rho_{object}}{\rho_{fluid}}$$

The Procedure

Measure the masses of the spheres, the metal cylinder and also the balloon. In the case of the balloon, I will not fill your balloon with helium until you show me the recorded mass of your balloon. You will also need to measure the volumes of the cylinder and also the balloon (after it is filled).

Your Standard values: You complete this table

Obtain the masses for these objects from the balance.

You will not need to calculate the density of the balloon here.

Measure for here with vernier caliper and precision scales.

Object	Mass [kg]	Volume [m^3]	Density [kg/m^3] $\frac{mass}{volume}$
cylinder		$V = \pi r^2 h$ =	
metal sphere		$V = \frac{4}{3}\pi\left(\frac{diameter}{2}\right)^3$ =	
ping pong ball		$V = \frac{4}{3}\pi\left(\frac{diameter}{2}\right)^3$ =	
wood sphere		$\frac{4}{3}\pi\left(\frac{diameter}{2}\right)^3$ =	
balloon			

First, let's make sure you have calculated your volume of your cylinder correctly.
The standard values for some metals are (in kg/m^3):

Aluminum: 2800 ,Copper (Cu): 8960, steel: 7870

Compare your density to the standard value for your metals to make sure your calculations are correct also it is helpful to note that 1g/cc is equal to $1000 kg/m^3$ and 1cc is $1x10^{-6} m^3$.

Note: some students have reported that it is better to obtain these values for the wooden spheres before doing the experiment because of the water that penetrates these items.

Experiment 1

In the first experiment, you will determine the experimental density of aluminum (with Archimedes' principle) and then compare it to your previously calculated value. To do this, attach thread to the **aluminum cylinder** and also to a spring scale. Place the aluminum cylinder into a graduated cylinder and measure the weight of the cylinder. Pour water into the cylinder until the aluminum cylinder is completely submerged and record the new weight of the aluminum cylinder from the spring scale. **It is a very important observation now that the effective weight of the**

cylinder decreases as water is filled into the graduated cylinder. Record these items below:

Weight of AL Cylinder in air [N]: $W = \dfrac{m[\text{grams}]}{1000\,\dfrac{\text{grams}}{\text{kilograms}}}\, g\left[\dfrac{m}{s^2}\right] =$ ____________________ N

Weight of AL Cylinder in water [N]: $W = \dfrac{m[\text{grams}]}{1000\,\dfrac{\text{grams}}{\text{kilograms}}}\, g\left[\dfrac{m}{s^2}\right] =$ ____________________ N

Since the AL cylinder is completely immersed in water, the volume of the fluid displaced is equal to the volume of the cylinder.

Volume of fluid displaced [m^3]: $V_{fluid} =$ ___________________ m^3

Buoyant force (calculated) [N]: $F_b = \rho_{water}\, g\, V_{fluid} =$ _____________________ N

The measured buoyant force is given by the difference in the weights of the cylinder when in air as compared to its apparent weight in water.

Buoyant Force (measured)[N]: $F_{b\ measured} = W_{cylinder\ in\ air} - W_{cylinder\ in\ water} =$ ____________ N

Theoretical buoyant force [N]: $F_b = \rho_{water}\, V_{object}\, g =$ _______________ N

Calculate the error: $\%\ error = 100\,\dfrac{F_{b\ theoretical} - F_{b\ measured}}{F_{b\ theoretical}} =$ _____________________

Experiment 2

I now want you to repeat the experiment that you just did but now with the metal sphere. You can tie a thread through the hole in the sphere.

Weight of Sphere in air [N]: $W = \dfrac{m[\text{grams}]}{1000\,\dfrac{\text{grams}}{\text{kilograms}}}\, g\left[\dfrac{m}{s^2}\right] =$ ____________________ N

Weight of Sphere in water [N]: $W = \dfrac{m[\text{grams}]}{1000\,\dfrac{\text{grams}}{\text{kilograms}}}\, g\left[\dfrac{m}{s^2}\right] =$ ____________________ N

Since the sphere is completely immersed in water, the volume of the fluid displaced is equal to the volume of the sphere.

Volume of fluid displaced [m^3]: $V_{fluid} =$ ___________________ m^3

Buoyant force (calculated) [N]: $F_b = \rho_{water}\, g\, V_{fluid} =$ _____________________ N

The measured buoyant force is given by the difference in the weights of the sphere when in air as compared to its apparent weight in water.

Buoyant Force (measured)[N]: $F_{b\ measured} = W_{sphere\ in\ air} - W_{sphere\ in\ water} =$ ____________ N

Theoretical buoyant force [N]: $F_b = \rho_{water}\, V_{object}\, g =$ _______________ N

Calculate the error: $\%\ error = 100\,\dfrac{F_{b\ theoretical} - F_{b\ measured}}{F_{b\ theoretical}} =$ _____________________

Experiment 3

When the spheres float in water you will not need to tie them to a string to determine the weight when floating (the buoyant force will equal the weight). Place the sphere into the cup and pour water into the cup until the top of the sphere is level with the top of the cup (a card across the top of the cup will help you determine how much water to put into the cup). Measure the distance (h) from to top of the cup to the water. The volume of the cap of a sphere of radius R (that portion above the water) is given by:

$$V_{cap} = \frac{1}{3}\pi h^2 (3R - h)$$

We can now find the density of the wooden sphere and compare it to the more exact value determined earlier. From Archimedes' principle, the sphere will float if the buoyant force is equal to the weight. Thus:

$$F_b = \rho_{sphere} V_{sphere} g$$

But the buoyant force is equal to the weight of the fluid displace which is given by:

$$F_b = \rho_{water} V_{sphere\ submerged} g$$

Since the two forces are equal we thus have:

$$\rho_{sphere} V_{sphere} g = \rho_{water} V_{sphere\ submerged} g$$

$$\Rightarrow \rho_{sphere} = \rho_{water} \frac{V_{sphere\ submerged}}{V_{sphere}} = \rho_{water} \frac{V_{sphere} - V_{cap}}{V_{sphere}} = \rho_{water} \left[1 - \frac{V_{cap}}{V_{sphere}}\right]$$

For the small and the large sphere, I want you to calculate the volume of the cap and also the volume of the sphere.

Sphere	Radius (m)	h(m)	Volume (m^3)	cap Volume (m^3)	V_{cap}/V_{sphere}
wood					
ping pong ball					

You can now calculate the experimental density of the wooden sphere. In each case, the density is given by:

$$\rho_{sphere} = \rho_{water} \left[1 - \frac{V_{cap}}{V_{sphere}}\right] : \%\ error = \frac{\rho_{standard} - \rho_{measured}}{\rho_{standard}} x\,100$$

Wood sphere: ρ=______________ kg/m^3: %error=____________

ping pong ball: ρ=______________ kg/m^3: %error=____________

Experiment 4

You are now going to verify Archimedes' principle for a helium balloon. Your results will have some error since the helium in the balloon is not pure.

Show me the recorded weight of your balloon and I will fill it with helium. We will measure the pressure using the simple water manometer that I have. The overpressure is given from the difference in levels (in m) of the water in the manometer.

$$\Delta P = \rho_{water}\, g[\Delta h] = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ Pa}$$

Thus the absolute pressure inside the balloon is given by the

$$P = P_{atm} + \Delta P = 101325 + \Delta P = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ Pa}$$

Note that in a more precise experiment, we would measure the atmospheric pressure.

Now tie a short thread to the balloon and determine the volume of the inflated balloon. This is done by placing the balloon into a box of peanuts. Fill the box level with the top by placing peanuts into the box. Remove the balloon. The empty part of the box is equal to the volume of the balloon. The volume of the balloon is given by the length x width x height of this **empty part** of the box.

$$V_{balloon} = \text{Length x Width x Depth} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ m}^3$$

Interestingly enough, you will also need to record the temperature in C which must be converted to K (T[K]=273.15+T[C]).

$$T = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ C} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ K}$$

We can now calculate the number of molecules of helium in the balloon by using the ideal gas equation of state.

$$N = [P + \Delta P]\frac{V_{balloon}}{kT} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ molecules}$$

where k is Boltzman's constant and is given by $k = 1.3806503 \text{x} 10^{-23} \frac{\text{Joules}}{\text{K}}$.

In order to find the mass of the helium inside the balloon, we first find the number of moles inside the balloon. This is given by

$$n = \frac{N}{N_A} = \frac{N}{6.02 \text{x} 10^{23}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ moles}$$

Assuming the gas inside the balloon is only helium, we can then obtain the **total mass** of the helium inside the balloon:

$m=4n$ (where 4 is the atomic mass of helium).

So $m=$________________g=______________________ kg

Now tie a weight hanger to the end of the thread and place about 10 g mass on the weight hanger. Weight the mass of the hanger and then allow the balloon to lift on the mass and measure it again. The difference in the masses is how much mass the balloon can lift. Record this value.

$$\Delta m = Mass_{\text{hanger w/o balloon}} - Mass_{\text{hanger with balloon}} = \_\_\_\_\_\_\_\_\_\_ \text{ g} = \_\_\_\_\_\_\_\_\_\_\_\_ \text{ kg}$$

The total amount of weight which the balloon can lift is then given by:

$$lift = [m_{balloon} + m_{helium} + \Delta m] g = \_\_\_\_\_\_\_\_\_\_\_\_ \text{ N}$$

Now we can determine the theoretical lift from Archimedes' principle. We will assume the density of air is about 1.2041 kg/m^3, although this is only approximate. In a more precise experiment, we would measure this also.

The theoretical buoyant force would then be:

$$F_b = \rho_{air} V_{balloon} g = \_\_\_\_\_\_\_\_\_\_\_\_ \text{ N}$$

Compare this to the measured lift:

$$\%difference = 100 \times \frac{F_b - lift}{\frac{1}{2}[F_b + lift]} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Your writeup for this lab consists of the normal abstract, procedures, this data sheet filled in with the results of each of the calculations for this lab which are contained in the spreadsheet and conclusions with references. Oh, and yes you can keep your balloon!