Simple Harmonic Oscillation
2019

In this lab, you will experiment with two simple harmonic oscillators, namely the simple pendulum and the spring – mass system. Let's go through the analysis of each of these systems.

In class notes, you will learn that if a force **F** is restoring (meaning that it points towards an equilibrium position or in the opposite direction of the displacement) and is linear in the displacement variable, then the motion is described by:

$$x(t) = A\cos(\omega t + \phi)$$
$$v(t) = -\omega A\sin(\omega t + \phi)$$
$$a(t) = -\omega^2 x(t)$$

In these equations, x represents a generalized position, v represents a generalized velocity and a represents a generalized acceleration. We can show that **for the spring mass system**, x can represent the correct position as a function of time if

$$\omega = \sqrt{\frac{k}{m}}$$

where k is the spring constant and m is the mass attached to the spring. For the simple pendulum, the condition for the angular frequency must be given by

$$\omega = \sqrt{\frac{g}{L}}$$

where g is the acceleration due to gravity and L is the length of the simple pendulum.

In practical terms it is much easier to measure either T (period) or if it is vibrating very fast, f (frequency). These quantities are all related to ω.
quite simply:

$$\omega = 2\pi f, \omega = 2\pi/T, \text{ and } T = 1/f \ .$$

This means that the period of the spring-mass system should be given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and the period of the simple pendulum should be given by

$$T = 2\pi\sqrt{\frac{L}{g}}.$$

Procedure

Part 1: The period of a spring-mass system.

A note on counting: say “zero” at the instant you release the mass.

a:

You will need to measure the spring constant of the spring provided to you first. Do this starting with 10 grams mass on the spring and increasing the mass (and thus the force) up to a maximum of **about 60 grams** in **at most 5 gram** increments. This is almost identical to a measurement you made in a previous lab (see lab 5) so this should go very quickly for you. There is a section of the spreadsheet which will help here. The slope of a graph of force vs. displacement will be your spring constant.

b:

Now that you have measured your spring constant, you are ready to measure the frequency of oscillation for your system.

Place about 20g on your spring. Time 20 oscillations and find the time per oscillation. Be aware that at the moment you release the mass and start the stop watch, you should say “zero”, not “one”.

Repeat this for 20g, 25g, 30g, 35g, and 40g masses.
For the spring mass system, we have:

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} \Rightarrow \frac{T}{2\pi} = \sqrt{\frac{m}{k}} \Rightarrow T^2 = \frac{4\pi^2}{k} m$$

If our development is correct for the spring-mass system, then you should be able to show by plotting a graph with T^2 on the y-axis and m on the x-axis that the slope of a line passing through these three points is equal to $4\pi^2/k$. Compare the spring constant obtained this way to that you measured earlier by use of the %error. You will note that my %error is large because my numbers are randomly picked. You will have a much better result.

c:

I want you to now investigate a different spring-mass system, namely the slinky. Since the mass suspended below the slinky holder is proportional to the number of turns, I want you to collect data regarding the period of the slinky vs the number of turns below the older. In fact, this is a more complicated analysis than it may appear at first: from a previous lab, you learned that the spring constant for two spring in series adds as $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}$. So as you take additional turns off of the slinky, the spring constant and also the mass are both not constant. In this analysis, you are only looking at the behavior without delving further into the theory.

Part 2: The period of a simple pendulum.

An important experimental note: the period of the simple pendulum depends upon having small angle oscillations. Keep your angles less than about 10^0.

a:

Choose one of the small metal balls provided (use the heavier one here). Connect enough **fishing line** to this mass so that you can vary your length between 0.5 m and 7 m. In the first part of this portion of the lab, you will verify that the period of a simple pendulum varies as $\sqrt{L}$ where L is the length of the pendulum. Previous year students have emphasized the importance of using fishing line for this first experiment.

Measure the length from your holder to the pendulum until you have a length of 0.5 m. Time 20 oscillations of your pendulum and find the time per oscillation. Repeat this for lengths of about [1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0]m keeping the mass constant. As the string gets longer, you will probably need to move out to the Derby Center steps. The simple way to do this is to get slightly more than 5 m of fishing line at the beginning.

If our development of the simple pendulum is correct, then you should be able to show by plotting a graph with T^2 on the y axis and L on the x axis that the slope of the line fit to these data points has a slope given by $4\pi^2/g$. Compare the slope of your graph to this theoretical value by use of the %error.

b:

Now, select another mass. For these parts of the lab, use thread since the weight of the thread is a consideration. Attach this to your pendulum and set the length to the center of mass to be 1.0 m. For each of these masses, leave the length the same and time 20 complete oscillations All together, you should have made three measurements at the length of 1.0 m and you should weigh each of the masses that you have used. If our formulation of the simple pendulum is correct, then the period should be independent of the mass of the pendulum. You can show this by plotting a graph with T on the y axis and m (in kg) on the x axis. A line passing through these points should have a slope of almost 0 (zero) meaning that the period of the simple pendulum is not dependent upon the mass of the simple pendulum. You probably won't get exactly zero here. You will want to think of some reasons that this is the case. Some hints about this: (1) the string actually does stretch and (2) you might not have exactly 1 m of string length to the center of mass. However, if you've done your experiment correctly here, you should be able to interpret the slope of your graph as showing almost no dependence upon mass for the period of the simple pendulum. Compare, for example, this slope to the previous slope. Repeat this for the other mass.

Write-up

In addition to the three graphs discussed above, your write up should discuss the two problems that are under consideration. From your data, you should be able to determine if the formulation is in agreement with reality. You should discuss possible sources of deviation in your results. You should also take this opportunity to make sure you completely understand the connection between frequency (f), period (T) and angular frequency (ω).