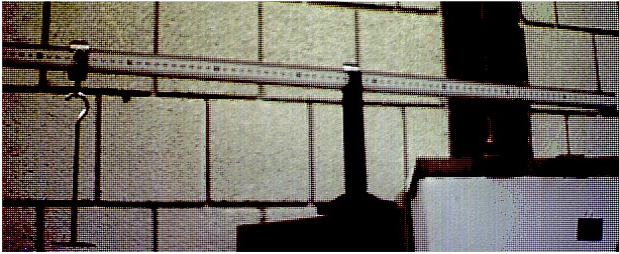


# Equilibrium of Rigid Bodies

revised Fall 2021



In this lab, we consider some of the consequences and conditions for static equilibrium. **The conditions for static mechanical equilibrium for a rigid body are**

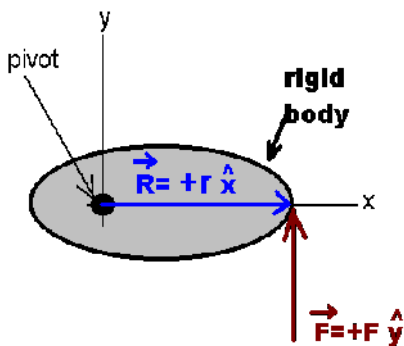
$$\sum \vec{F} = \vec{0} \text{ and } \sum \vec{\Gamma} = \vec{0}.$$

You are, by now, quite familiar with the first of these conditions and have worked with this in an earlier lab or two. The second condition needs some explanation.

A torque ( $\Gamma$  or  $\tau$  is the usual symbol used to represent this) is a force acting at a distance on a body. The torque is a vector which comes about in an unusual manner (and is actually a different type of vector). The torque acting on a body is defined as the cross product of the radius at which the force is acting and the force which is acting:

$$\vec{\Gamma} = \vec{R} \times \vec{F}.$$

Let's consider a simple situation. Suppose the force is acting upon a body at a distance  $r$  from a pivot point as shown below. In this simple example,  $\vec{F} = |\vec{F}|\hat{j}$  and  $\vec{R} = |\vec{R}|\hat{i}$ . The torque can then be calculated by using the determinate:



$$\vec{R} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ R & 0 & 0 \\ 0 & F & 0 \end{vmatrix} = \hat{i}(0) + \hat{j}(0) + \hat{k}(RF).$$

In this simple example, the torque would be given by

$$\vec{\Gamma} = RF \hat{k}.$$

We can also determine the magnitude of the torque by the definition of the cross product. The magnitude of the cross product of two vectors  $\vec{A}$  and  $\vec{B}$  is given by

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin(\theta)$$

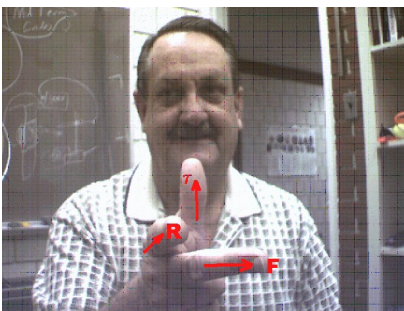
where  $\theta$  is the angle between vectors  $\vec{A}$  and  $\vec{B}$ . Interestingly enough, we can use this definition together with the cross product which comes by taking the determinate of the two vectors to obtain the angle between

the two vectors:  $\sin(\theta) = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}$ . This can be combined with

the dot product to give:

$$\tan(\theta) = \frac{|\vec{A} \times \vec{B}|}{\vec{A} \cdot \vec{B}}.$$

There are many ways that have been developed to obtain the cross product using your right hand (these rules are known as "right hand rules"). Essentially, if you align your forefinger along the direction of  $R$ , and point your middle finger along the direction of  $F$  then your thumb points along the direction of the torque as shown. Check this with the previous example: align  $F$  along  $+y$ ,  $R$  along  $+x$  then the torque will lie along  $+z$ . Ultimately these come about because our math uses a right-handed coordinate system in which  $\hat{x} \times \hat{y} = \hat{z}$  and the requirement that the volume of the elementary parallelepiped which is given by  $\text{volume} = (\hat{x} \times \hat{y}) \cdot \hat{z}$  be a positive quantity.



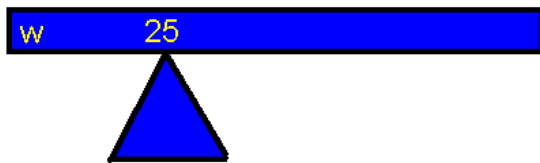
## Terminology Note

Important note on my terminology: pivot point is the point that something like a pivot is touching a body (it would, for example, be where the point of the triangle below touches the meter stick) and is a physical object in touch with another physical object. Axis is an arbitrary location about which torques are located. It can also be at the pivot point but this is not a necessity.

## Sign Convention for torques

We also use a convention for the sign of the torque. If a torque by itself would make a rotation in the clockwise direction about an axis, this is a negative torque. If a torque by itself would give a counterclockwise rotation about an axis, this is a positive torque. As you now know from the previous discussion, the positive or negative only refers to whether the torque lies along the positive z axis or the negative z axis. ***In addition, a positive torque about one axis may become a negative torque about another axis.***

**Read the previous sentence again please!**



**An example:** Let's do an example to show that in equilibrium, the conditions for equilibrium are met. Consider a meter stick which is pivoted at the 25 cm mark which weighs 0.1 kg. Where would a 0.5 kg weight need to be placed in order for the meter

stick to balance on the pivot?

If the meter stick weighs 0.1 kg, then since it is in static equilibrium, we require  $\sum \vec{F}_{\text{ext}} = \vec{0}$ . We first find the force exerted by the pivot on the meter stick.

$$\sum \vec{F}_{\text{ext}} = \vec{0} \Rightarrow F_{\text{pivot}} - m_1 g - m_2 g = 0 \Rightarrow F_{\text{pivot}} = (m_1 + m_2) g$$

In this example, this force is given by:

$$F_{\text{pivot}} = (0.1 + 0.5) 9.8 = 5.88 \text{ N}$$

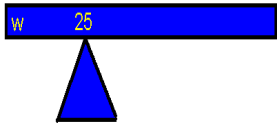
Now calculate the sum of the external torques acting on the meter stick about an axis. I am going to choose the pivot point to be my axis although this is not a necessary choice if the meter stick is in equilibrium.

The torques that you have here are the following:

- (1) The weight of the meter stick acts at the center of mass of the meter stick.
- (2) The 0.5 kg weight acts at the position that it is placed.
- (3) The pivot acts at the pivot point.

**Students** will often neglect the torque contributed by the pivot point and do not understand why the answers do not provide a total external torque of zero. Students also often neglect the center of mass of a meter stick. Don't be one of these students.

**Read the previous three sentences again!**



(1) The pivot was placed at  $x=0.25$  m. The center of mass of the meter stick is at  $x=0.5$  m. Thus, regarding the axis as being at the pivot point, we see that the torque from the meter stick is acting at a distance of 0.25 m from the axis.

- (a) Looking at the picture, draw in this force and show its distance from the axis.  
 (b) Now ask yourself the question: does this produce a clockwise or a counter clockwise rotation about the axis? The answer is clockwise. Thus this torque is negative.  
 (c) The value of the torque is then given by:

$$\Gamma = -(0.25\text{m})(0.1\text{kg})(9.8 \frac{\text{m}}{\text{s}^2}) = -0.245 \text{ Nm.}$$

(2) The 0.5 kg weight acts at some (unknown)  $x$  position. It is at a distance of  $[x-0.25\text{m}]$  away from the axis.

(a) Looking at the picture, draw in this force and show its distance from the axis. Use your intuition in making this sketch. It does not need to be perfectly located but it does need to be on the proper side of your pivot.

(b) Now ask yourself the question: does this produce a clockwise or a counter clockwise rotation about the axis? The answer, if you have done things correctly is counter clockwise. Thus this torque is positive.

(c) The value of the torque is then given by:

$$\Gamma = + (|x-.25|\text{m})(0.5\text{kg})(9.8 \frac{\text{m}}{\text{s}^2}) = + 4.9|x-.25|\text{Nm.}$$

(I've used absolute value signs to make sure things stay of the proper sign)

(3) The pivot exerts an upward force on the meter stick given by 5.88N. It is at a distance of 0 m from the axis.

(a) looking at the picture, draw in this force and show its distance from the axis.

(b) Now ask yourself the question about the sign of this torque. (here the answer is that it is neither positive nor negative. (see (c) below).

(c) The value of this torque is zero since it is acting through the axis.

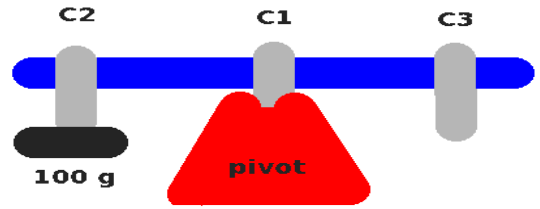
Now solve for the position:

$$\sum \Gamma = 0 \Rightarrow -0.245 \text{ Nm} + 4.9|x-.25|\text{Nm} + 0\text{Nm} = 0 \Rightarrow x - 0.25 = 0.05 \Rightarrow x = 0.2 \text{ m.}$$

So you would need to place the weight at a distance of 0.2 m away from the pivot.

## The Experiments

**Experiment 1:** Take three clamps. Weigh each of the clamps and record the weight of each. You will want to keep track of this weight. Perhaps you might name your clamps.



Weigh your meter stick. Attach one of the three clamps (C1) to your meter stick at a position close to the 50 cm mark **with the screw pointing downward**. Weigh two weight hangers and record their weights. You will also want to keep track of these weights.

Adjust the center clamp (C1) on the meter stick until the stick balances on the knife-edge stand. Record the clamp position as  $x_1$ . This is **Experiment 0** on the helper.

Place clamp (C2) at the 15-cm position [ $X_2=0.15$ ] (15 cm from 0) with the screw pointing downward. Place on (C2) a mass hanger [H2] and a mass [M2]=100g. Record the actual weights on C2.

Place clamp (C3) on the meter stick, together with a hanger [H3] and a mass [M3] of 200g.

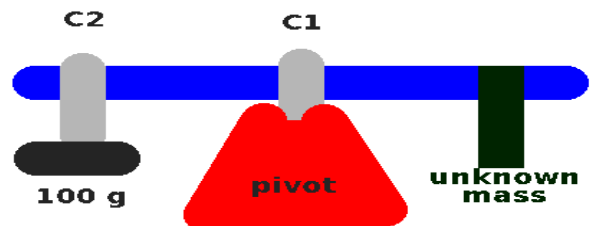
Move (C3) until you establish the condition for equilibrium. Record your distances.

**Compute the torques about the pivot point** and find the percent deviation in the computed values (this means, add up all your positive torques(+ $\tau$ ) and your negative torques(- $\tau$ .) You will want to use the formula for the percent deviation below:

$$\% \text{ deviation} = 100 \times \frac{\sum \text{signed torques}}{\sum \text{absolute value of all torques}}$$

because we are trying to compare something that is ideally zero to what it should be which is zero: that makes for a different need for an error criteria here.

**Experiment 2:** Consider your unknown mass to be the mass hanger, the meter stick clamp (C3) and the ring stand clamps. Hang the ring stand clamp from the hanger. Change the position of your unknown mass until equilibrium is established. Use the condition for equilibrium to determine the mass of your unknown mass. Do this by varying cell D17 on the helper until cell E20 is less that .1 %.



**Hint:** In your spreadsheet for experiment 2, vary the value of the unknown mass using the purple scroll bar until you obtain the smallest percentage deviation between + and - torques. I have marked important areas using a red font on the spreadsheet. Your spreadsheet will automatically subtract the mass of clamp 3 and

Unknown Mass [g]	Meter stick mass [g]
0.00	103.51
Vary this unknown mass To make minimum % deviation (cell E21)	abs(% deviation of $\Sigma\tau$ ) 0.0000

Your spreadsheet will automatically subtract the mass of clamp 3 and

give the actual value of the unknown mass. The image above shows the location where you manually fit to find the unknown mass for experiment 2. **You will need to have set the signs of your torques (-, 0 or +) with the blue scroll bars before you can solve for the unknown mass. The image below shows you how to use the blue scroll bars for this. Slide the button for the desired sign of the torque.**

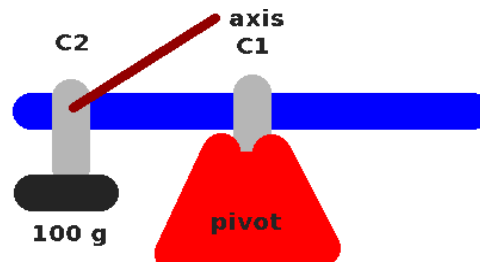


## After this:

Weigh your unknown mass (which included clamp C3, the hanger and the ring stand clamp) and determine the percentage error in your measurement. We'll assume the scale gives the correct mass reading here.

*Note: your experimental setup is exactly the same for experiment 3, 4a and 4b.*

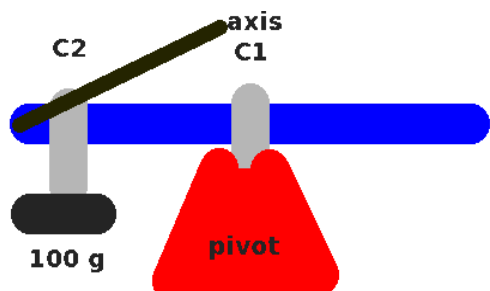
**Experiment 3:** Remove all clamps, masses and hangers except for C1. Place a clamp C2 on your meter stick near the end. Place hanger H2 on C2 and place a mass  $M_2=100\text{ g}$  on the hanger.



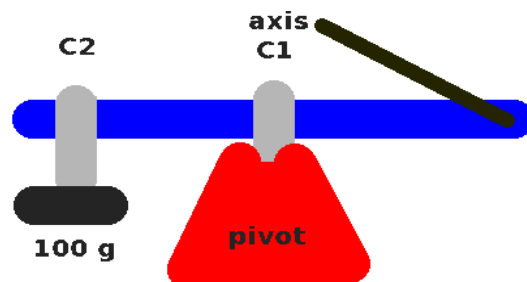
Move the meter stick in the support clamp (C1) until the system is in equilibrium. Record the support position  $X_1$ .

Now let's do something unusual. Regard your axis as being at the position of clamp C2. Calculate the sum of the torques again and do your analysis as you did in experiment 1. (It is a bit easier this time). **Note: this is partially an exercise to make sure you can get signs right.**

**Experiment 4:** Leave your system in equilibrium as in Experiment 3. Calculate the net external torque about each end of the meter stick.



**Experiment 4a.**



**Experiment 4b.**

As in experiment 3, **you will need to make absolutely sure you can obtain the correct signs with your torque.** Now, think about this: comparing experiment 3, 4a and 4b: **you will not necessarily see the same deviations each time. Why is this?** (This is an important question for you to write about).

## **Writeup**

Your writeup for this lab should include a discussion of the conditions for static equilibrium and sample calculations showing how you calculated the torque about the pivot point for each of the experiments. You should make absolutely sure that you understand (before leaving lab) how to determine torques, directions and to work problems involving torques and static equilibrium. And answer the question above in experiment 4. Don't forget to include proper references, sketches and sample calculations.