Centripetal Forces & Hooke's Law

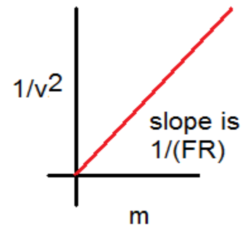
revised Fall 2021

Note: Safety glasses are required to be worn while anyone is working today.

In today's lab, you will use the Welch Scientific Company's Centripetal Force Apparatus to verify the relationship between centripetal force, mass, velocity and radius. You may recall from class that for a mass moving in uniform circular motion, the centripetal force is given by:

$$F = m\frac{v^2}{R}$$

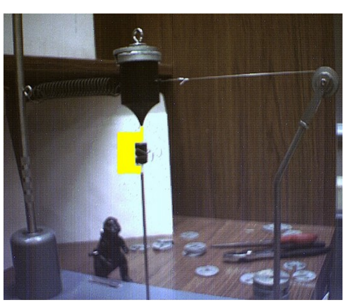
where F is the centripetal force, m is the mass undergoing circular motion, R is the radius of the circular orbit and v is the tangential velocity of mass m.



A theoretical plot of the mass versus $1/v^2$ would provide a straight line with a slope which is equal to (1/FR) if both the force and radius are constant. For this portion of the experiment, both F and R will be constant: the force is equal to the amount of force required to hold the pendulum over the paper tab and this occurs at the same radius each time.

You will explore this for several different values of m in this lab, namely the mass of the pendulum which I call m_p . You will (by screwing masses onto the top of the pendulum) investigate this for $m_p, m_p+50\,g, m_p+100\,g, m_p+150g$.

Experimental note: when you run your experiments, the upper most screw must be tight and the correct position for the pendulum is over top of the indicator bar with the spring, string, bar, and stand **making a square**. If these two things are not done, you will have larger experimental errors. Also make sure you record the time for 20 revolutions in seconds. In particular, be sure to include that 60s = 1 min if your time runs over 1 minute! Also as far as obtaining data is concerned, practice with a few turns until you hit the tab each revolution. Start your timer when you are ready. Start the watch with strike 0 (time 0), strike 1 (time T) , … , strike 20 (time T_{20}). In other words when counting revolutions, start at zero.



We could also vary the radius of orbit but this proves to be significantly more difficult so we will keep R fixed in this lab. The image to the left shows the apparatus.

There is a small tab of paper or plastic attached to the upward sticking rod. When the apparatus is spinning at the correct velocity so that the pendulum mass is aligned over the metal indicator, and the mass is spinning, you will hear a click as the mass strikes the paper tab as shown in the image to the left.

Procedure: For each of the 4 masses, you should find the time T_n that it takes the system to spin through n revolutions. The period (T) which is the amount of time required for 1 revolution is then given by

$$T = \frac{T_n}{n} \quad .$$

Here, n should be at least 20, but you can do more if you are so inclined. Remember, T is the amount of time for one revolution. Note that you will use your smart phones to do this timing.

Next, you should measure the distance from the center to the tab of paper to the center of the upright rod. This will be R. If you have a less than perfect tab of paper, let me know. Be sure to convert this measurement into meters. Now, in order to find out how much force is required to move the mass to the tab of paper, use the weight hanger together with the special attachment (the "paper clip leash") which I have provided (you can see this in the first image). Add weight as shown in the first image until the pendulum is aligned with the tab of paper. **Under no circumstances will the pendulum be rotating during this measurement; otherwise you will have less than perfect results and a probable head whack.**

A question now: what are you measuring when you measure this particular force which is given by:

$$F = mg \quad ?$$

The answer is that you are measuring how much force the spring is exerting on the pendulum while it is swinging trying to pull the mass backward towards the center. Of course, you’re not going to want to be spinning the thing until you remove the paperclip leash that I’ve made for you!

Now the next thing is this: How do you know what the tangential velocity is that you are measuring? And here is the answer: in a period T, the mass travels through a distance given by:

$$s = 2\pi R$$

Thus you are able to fine the tangential velocity easily since it is given by:

$$v_t = \frac{2\pi R}{T} \quad ,$$

where R and T are values that you have measured.

Important experimental notes

Here are important notes to reduce your errors: when the spring is removed from the pendulum, the pendulum should hang right over the rod that is sticking up. You will need to adjust the screw on top to make sure this is the case. Failure to do this will produce larger errors. After reconnecting the spring to the pendulum, you will want to balance the system by moving the adjustable mass on top. The system is balanced when it rotates without jumping around but the spring is attached to the pendulum while testing this. You will also want to make sure the system is leveled by adjusting the legs on the board. There are lots of apps for your smartphones that can work as a level for this that you can use.

Analysis: The centripetal force is related to the mass, velocity and radius by the first equation above. For each of your measurements, calculate the experimental centripetal force. Compare these calculations to your measured centripetal force by using percent error. You may consider that the correct value for the centripetal force is obtained by the masses attached to the hanger.

In this lab, you will also want to plot a graph of m vs. $(1/v^2)$. Look at the theoretical equation for the centripetal force, which is:

$$F_{centripetal} = m\frac{v^2}{R}$$

where v will be a tangential velocity. Now define a variable

$$y \equiv \frac{1}{v^2} \quad .$$

Then in terms of y we have:

$$F_{centripetal} = m\frac{1}{yR} \quad .$$

Solve this for y:

$$y = \frac{1}{RF}m$$

Now the equation for a straight line is given by:

$$y = \text{slope} \cdot x + \text{intercept} \quad .$$

By comparison, if $\text{mass} \equiv x$ and $\text{slope} \equiv \frac{1}{RF}$ you would be able to measure the product of RF.

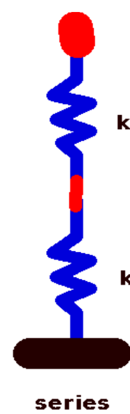
These measured values can be compared to the value of R and F which was measured previously to give a percentage error. Compare your slope to the theoretical value of F that is measured by hanging weights over the end of it as shown in the first image. In principle, you should obtain fairly good agreement here. Discuss in your write-up where experimental errors may have entered into the results. There may be a small y-intercept which is a reflection of experimental error.

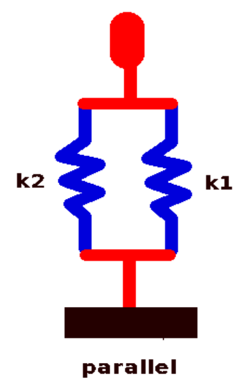
Part II: Hooke's Law

In this part of the lab, do not stretch the silver springs beyond *about* 10 cm. You should wear eye protection during this part of the lab. As you have seen, springs are very useful for measurement of force. You may recall from class that to first order, the force required to stretch a spring is related to the distance the spring is stretched by Hooke's law:

$$|\vec{F}_{spring}|=k|\Delta\vec{x}|$$

where Δx is the displacement from equilibrium and k is the spring constant (ignoring the negative sign). I would like for you to learn how to quickly measure the spring constant of springs. Although you will not be using this more in today's lab, it is an important technique that you will use in at least one later lab. You will need to construct several setups for the measurement of the spring constant for the two springs provided. This is so you can compare one of your other springs to this system and see if the spring constants for springs in parallel add or if the inverses add. **You must answer the following questions based upon your data**: How do **series** springs add: as $\frac{1}{k_{eff}}=\frac{1}{k_1}+\frac{1}{k_2}$ or do they add as: $k_{eff}=k_1+k_2$? Answer the same question for **parallel** springs. Answer these for both the series and parallel combination, **based upon your data**. **Make sure to show your math here in your lab writeup!** Note the sketches of parallel vs. series combinations shown. **An important note: replace the springs into their proper package after use.**





In today's lab, I want you to measure the spring constant of the 2 silver springs, and also for the series and parallel combinations of the two springs and also for the slinky.

In the past, students have found the simplest way to do this is to place the 100 cm end the meter stick on the floor, and lower the spring holder until it is at a convenient location. Measure from the bottom of the small hanger; **the first measurement does not need to be zero on the meterstick and you also do not need to have zero mass for the first reading.** We are only interested in the slope of the force vs. displacement graph. The analysis is this: let M be the total mass attached to the spring. Then the applied force is F=Mg. If Hooke's law is obeyed then:

$$F=Mg=kx-kx_0\Rightarrow slope=k\quad .$$

A plot of x on the x-axis and F=Mg on the y-axis will produce a straight line if Hooke's law is followed. **Note that the suggested weights are only suggestions. Be careful not to overstretch your spring.**

You will need to make a data table containing m, F and x in SI units for the measurements. For 5 different masses, record the position of the bottom of the hanger attached to the spring and then for each spring, plot a graph of x vs. F. Other than performing a good experiment here and determining your spring constants, no further analysis is required for part II except for the determination of how spring constants add mentioned above. However, you will most certainly want to write about this in your lab writeup. You will also want to remember how to do this for future reference since in a future lab, you will have to determine the spring constant.

You will note that the spreadsheet reports the R^2 coefficient. You can find this out by reference to the Open Office wiki[1] that this is the Pearson Coefficient and is determined by the following where x is the fit and y is the measured data:

$$R^2 = \left[\frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})} \sqrt{\sum_{i=1}^{n} (y_i - \bar{y})}} \right]^2 : \bar{x} \equiv \frac{\sum_{i=1}^{n} x_i}{n} : \bar{y} \equiv \frac{\sum_{i=1}^{n} y_i}{n} \quad .$$

A value of $R^2=1$ indicates complete correlation while a value of 0 shows no correlation.

When you measure the spring constant of the slinky, do it in the following way: find the mass of the slinky, count the number of turns on the slinky and then calculate the mass per turn by dividing the two. With a paper board, you can then hang 5 turns, 10 turns ,etc. Note that to get the mass hanging, you multiply the mass per turn by the number of turns hanging, and measure the hanging length each time. Plot your results on a graph with force on the x-axis and length on the y axis and provide a qualitative discussion of the results. Note that departures from a linear behavior indicate that this spring does not obey Hooke's law.

[1] https://wiki.openoffice.org/wiki/Documentation/How_Tos/Calc:_PEARSON_function