Energy and power

Revised Fall 2021

Energy, work and power in physics are defined. Work is a force exerted (on matter) which is accompanied by a displacement along the direction of the force. By the work-energy theorem, work produces a change in energy. Power is the rate of doing work.

The first portion of this lab involves simply calculating the work done to walk up a flight of stairs. One comment regarding notation: in the following notes, s represents the displacement. I am defining the work done on a system by an external entity which is exerting a force $\vec{F}$ on the system while the system is undergoing a displacement $\Delta\vec{S}$ in the direction of the applied force as positive. In the analysis below, I will try to be painfully clear about the various signs involved.

Procedure:

Note: the spread sheet for this portion is WorkPower01

When you walk up stairs, you do work against the gravitational force by exerting a force on a step. The force (Note: it is in the negative y direction) which you exert to increase your height is given by:

$$\vec{F}_{\text{on step}} = m\vec{g} = -mg\hat{y}$$

where m is your mass and $g = +9.8\frac{m}{s^2}$ which is the **magnitude** of the **vector** acceleration due to gravity. This required force is slightly higher than this in order to ascend (or slightly lower in order to descend). In the SI system, the mass is in units of Kg. The step accelerates in the -y direction since, according to Newton's law, the acceleration is in the direction of the force. This acceleration is very small because of the large mass of the earth. The step, according to Newton's third law, exerts an equal and opposite force on you (in the positive y direction) and is given by:

$$\vec{F}_{\text{from step}} = -m\vec{g} = +mg\hat{y} \quad .$$

The **step** does an amount of work on your body which is given by:

$$W = \vec{F}\cdot(\Delta\vec{y}) = (-(-mg\hat{y}))\cdot(\vec{y}_{\text{final}} - \vec{y}_{\text{initial}}).$$

If $(\vec{y}_{\text{final}} - \vec{y}_{\text{initial}}) > 0$,in which case you are climbing the stairs, then the work done on your body would be positive, resulting in an increase of your potential energy. If $(\vec{y}_{\text{final}} - \vec{y}_{\text{initial}}) < 0$, in which case you are descending, the work done on your body would be negative, resulting in a decrease of your potential energy.

In general, note that the work on a body **due to a force from an external agent** is:

Non-calculus	Calculus
$W = \sum_{\text{path}} \vec{F}\cdot\Delta\vec{s}_i$ i is a segment over which F is constant	$W = \oint_{\text{path}} \vec{F}\cdot d\vec{s}$

If the force is conservative, a class of forces including gravitational forces and electrostatic forces, then the work done is independent of the path. If the force is non-conservative, of which friction is an example, then the work done against non-conservative forces depends upon the path. For our purposes today, we will consider that the body is only working against the conservative force of gravity. In this case, the work required reduces to:

Non-calculus	Calculus
$W=\sum \vec{F}\cdot(\Delta \vec{x}_i)$ i is a segment over which F is constant	$W=\int_{x_{initial}}^{x_{final}} \vec{F}\cdot d\vec{x}$

Both forms become very simple in the additional case of a constant force:

$$W=\vec{F}\cdot(\Delta\vec{s}) \quad .$$

Notice that work is a scalar while force and displacement are both vectors. This is important in understanding that the step may do positive work or negative work on your body, depending upon the sign of the displacement vector, $\Delta\vec{s}$. Also note that work is not a conserved quantity: **there is no such thing as ΔW**.

By the work-energy theorem, this work results in a change in energy. In the case of walking up stairs, what has changed is potential energy. Thus the change in potential energy would be given by:

$$\Delta U=W=mg(\Delta y) \quad .$$

Work, potential energy (and kinetic energy) have units of Joules (J) in the SI system. The power exerted by the external agent in a time Δt is the rate of doing work which is:

$$Power=\frac{\Delta U}{\Delta t}=\frac{W}{\Delta t} \quad .$$

In the SI system, power has units of Watts (W). Be sure not to confuse Work (W) with the SI unit Watts (W). Each is understood in the context of usage.

Later in the class, we will discuss impulse, however it is convenient to introduce this now. It is also possible to calculate the impulse delivered. The impulse (SI units: Newton Seconds (N s)) is defined as

$$\vec{J}=\vec{F}_{average}\Delta t \quad \text{(non-calculus)} \quad \text{or} \quad \vec{J}=\int\vec{F}\,dt \quad \text{(calculus)}.$$

Both Joules and impulse have the same symbol but the symbol has different meanings. Again, you must understand the symbol in the context of usage.

When your body does work, this comes at the expense of electrostatic energy stored in the form of chemical bonds. While it is easy to calculate the energy expenditures in climbing stairs, it is not so easy to calculate this when descending. In our lab today, we will ignore the descending calculations.

Procedure

The spreadsheet calculations here are in WorkPowerEnergy.

Measure the height of a step, then count the number of steps from the basement of the Derby Center to the second floor. Weigh yourself (before and after). Walk up and down the steps at a constant rate, using a stopwatch to time yourself. Do this for four different trip times and calculate the power expended in Watts for each trip. **If your graph shows a sudden funny behavior**, I have found that students resolve this by remembering that a time reading on the stopwatch of 1:20, for example, means 80 s.

The energy which must be expended in walking to the second floor is given by:

U=mgh=mg(# steps X height of 1 step)

Height of a step (m)	h	
# steps total to 2nd floor	#	
Body mass (kg)	m	
Energy expended in ascent	U=mg(#h)	

Power calculations

Trip number	Time (s) for trip up stairs	Power (U/t) expended (Watts)
1		
2		
3		
4		

Analysis

Your analysis for this portion of the lab is completed by understanding how to do the appropriate calculations to complete the tables above. Be sure to read time correctly, especially if your graph shows sudden discontinuities. A similar method is used to calculate metabolism.

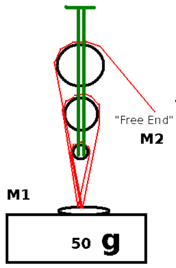
Mechanical Advantage

The spreadsheet to calculate this portion of the lab is **MechanicalAdvantage1**.
You should measure the tensions at the same time as you measure Δx_1 as described below.

One very important observation for this section is the following: With the string looping through the hook on mass m_1, when the string is pulled, complete the following table for the 3 situations when the free end (the end connected to m2) of the string is pulled through a distance of 0.05 m. Note that if you have a mass m_2 on the free end, this part of the experiment proceeds the same, it's just not "free".

Number of strings connected to mass m1 N	mass: m1 change in height (cm) Δx_1	Free end: (the end connected to m2) change in position (cm) Δx_2
1		-5
3		-5
5		-5

How to find Δx_1 and Δx_2



In the image, there is a hanging mass (the (about) 50 g mass) which I am calling 1 here. Make a mark on the string from the other end. Set a meter stick on the floor. When the mark on the other end of the string is pulled through a distance enough to lift mass m_1 by 10 cm , measure how far the mass (m_2) lowers by measuring from a position on the weight hanger holding the mass. If energy is conserved, you expect that

$$|m_1 \Delta x_1| = |m_2 \Delta x_2| \quad .$$

You will measure the tension in the free end by hanging a mass hanger on it and placing weights on it until (a)M_1 does not fall down (giving "**m_2 minimum**") and then (b) by placing more weights on m_2 until the mass M_1 starts to raise up (giving "**m_2 maximum**").

Verify that the system conserves energy

In an ideal world (no friction in the pulleys, for example), we expect that the amount of energy we put into the system (in the form of work with will be given here by

$$W_{in} = T \Delta x_2$$

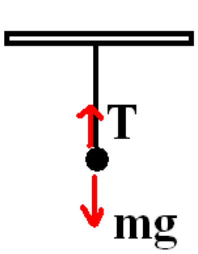
will be equal to the amount of work that the system does, which is given by

$$W = N T_m \Delta x$$

where the the tensions are T=tension in the free end) and T_m =tension in any one string holding up the mass.

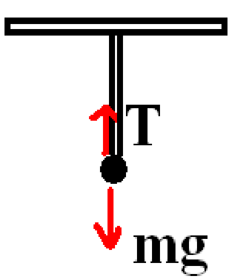
The fact that the strings are not exactly straight up and down also makes a slight inaccuracy here.. The spreadsheet will calculate a % difference between the two calculated works using the average m_2. We could calculate these tensions by finding the angles that the strings make. However assuming that the angles are fairly small here sidesteps that problem.

Analysis and understanding



Consider the following problem: A point mass (m) is held up by a string. What is the tension in the string?

Clearly, the tension is T=mg.



Now double the string in a funny way: imagine cutting it and attaching it to the mass as shown. What is the tension in each segment now?

Clearly, the total tension in each string must add up to the total weight being suspended. The tension is equally divided between the two segments. Thus, the tension in each string is now T=1/2 mg.

I believe now it is easy to see that if this process is continued you can see that the tension in each segment when there are N total segments would be given by:

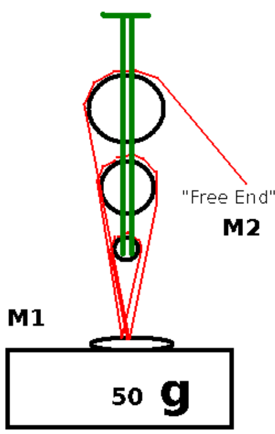
$$T = \frac{mg}{N} \quad .$$

Now suppose that one of the segments of string were cut and you were required to provide the tension that the upper holder was previously providing. How much force would you need to supply if there were N segments? Again the answer is:

$$T = \frac{mg}{N}$$

This is the basis for understanding mechanical advantage and we'll experiment with it using pulleys to verify this behavior. The ultimate consequence of mechanical advantage is that it takes much less force to hold up or lift an object than the actual weight of the object provided that your rope and pulley system is designed correctly. A correct design in a world filled with friction pretty much requires that each rope segment be wound over a pulley. We will modify this slightly. **However, you really already know this since you know that to make a rope twice as strong you double it.**

Procedure



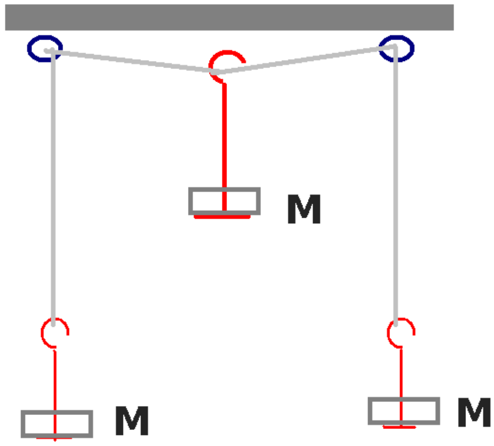
Your mass-pulley system should look something like what I have shown below. My experience says that you will probably need to watch an example of how to thread your strings to make this work.

Follow the instructions given earlier for finding m_2 maximum and m_2 minimum. Also find Δx_2 for each of the 3 loops when Δx_1=5 cm. **Note: 1 refers to the (about) 50g mass and 2 refers to the free end of the string. Also note that you measure Δx by measurements from the bottom of the mass hanger. You have a total of 2 numbers for each measurement. Subtract the two numbers for Δx. This gives then Δx_1 and Δx_2. If energy is conserved, then**

$$F_1 \Delta x_1 = F_2 \Delta x_2$$

which could be said "work in = work out." You will raise M1 by 10 cm each time for 1 loop, 3 loops and 5 loops, measuring how much M2 lowers each time.

The calculations for this, and also that the work in is equal to the work out (which it will not be due to friction) are in the spreadsheet named MechanicalAdvantage1. Your results will not be exactly correct, though, for other reasons as is shown, in part, below where the angle the string make has a strong influence on the tension in the string.

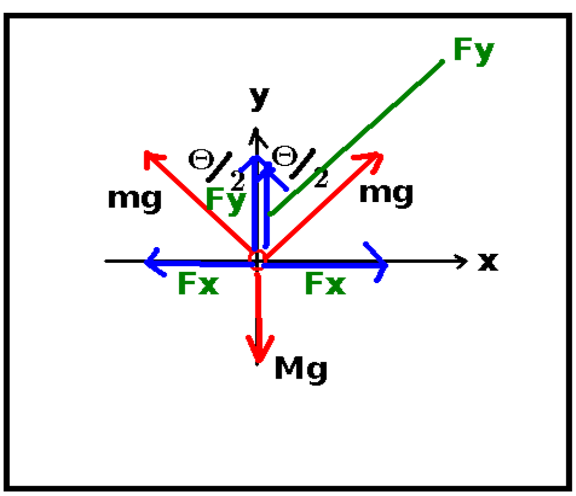


A more complicated simple machine

The spreadsheet calculations here are contained in MechanicalAdvantage2. **Although in the image, I show weight hangers with attached masses, in fact all you will need are masses which are identical** and we will calibrate this in terms of those units of mass. The masses are simply large nuts that are identical. **Hence, the mass unit is NMU (nut mass unit).**
Arrange your system as shown. The large hanger is in the center initially and at the top. Allow the central mass to fall until the system stops moving. Measure the angle Θ. The strings are thread and to achieve equilibrium, raise the central mass and then drop it.

You do not need to weigh the masses here. . If the weights are the same, then the system will be highly symmetric. **(You can test this for yourself by putting 1 NMU on the central mass and perhaps a 5 g hanger and dropping the mass again). You may need to rearrange the spacing between the pulleys for this to work. An analysis of the forces present with the angles measured would actually permit a calibration of the weight of 1 NMU in terms of g.** Doing so, however, is not part of today's lab. Note also that if you put 2 NMUs on the center, the analysis fails.

We want to analyze the forces present. A free body diagram for the system is shown below. Note that this is your first problem from the field of "statics."



The analysis:

$$\sum \vec{F} = m\vec{a}$$

$$X: mg\sin\left(\frac{\theta}{2}\right) - mg\sin\left(\frac{\theta}{2}\right) = 0$$

$$Y: 2mg\cos\left(\frac{\theta}{2}\right) - Mg = 0$$

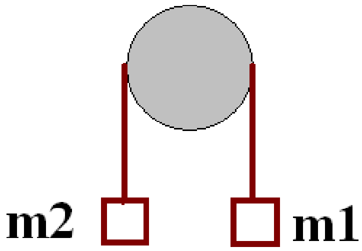
$$\Rightarrow \cos\left(\frac{\theta}{2}\right) = \frac{M}{2m} \Rightarrow \theta = 2\cos^{-1}\left(\frac{M}{2m}\right)$$

In particular, if M and m are the same, then the angle is given by:

$$\theta = 2\cos^{-1}\left(\frac{1}{2}\right) = 2 \times 60^0 = 120^0.$$

Analysis is provided in the spreadsheet for this calculation named MechanicalAdvantage2. You should measure the angle, M, and m (in terms of NMUs ... you do not need to weigh the nuts) in order to complete this portion of the lab. Friction will, of course, distort the results some so do not expect perfect results here. I found, however, that the results were quite close.

Atwood's machine



In class, we have seen how Atwood's machine works, and have done the analysis on it. If you will recall, Atwood's machine consists of a single pulley with two masses. **I recommend using the NMUs here for simplicity.**

From the analysis in class assuming m_1 is greater than m_2, you can determine the acceleration of this system which is given by:

$$a=\frac{m_1-m_2}{m_1+m_2}g$$

In previous versions of this lab, Atwood's machine was used to measure the value of g but errors associated with friction, the inertial of the pulley, and timing prevent very good measurements from resulting. In a later lab, we will later be able to measure g with the simple pendulum. Today, however, I want you to construct Atwood's machine and observe several situations only. In particular, I want you to observe what happens when the two masses are equal, and I also want you to observe what happens when m_1 is larger than m_2 and also what happens when m_2 is larger than m_1. In particular, you should note the amount of time required for the mass to move through a specified distance and relate this to the acceleration. **Your analysis on this part is permitted to be in the form of a narrative detailing the analysis (from class) of the Atwood's machine and also your observations.**