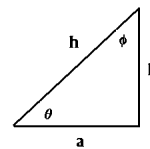


(1) Which of the following equations are dimensionally correct? Note: the following quantities have the following dimensions:

$$F: \left[\frac{ML}{t^2} \right]; m: [M]; a: \left[\frac{L}{t^2} \right]; t: [t]; x: [L]; E: \left[\frac{ML^2}{t^2} \right]; v: \left[\frac{L}{t} \right]; s: [L]$$

- (a) $F=ma$
- (b) $X=(1/2)at^3$
- (c) $E=(1/2)mv$
- (d) $E=mv$
- (e) $V=[Fs/m]^{1/2}$

(2) In the right triangle shown, find the following: h in terms of a and b . Then find $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$, $\sin(\phi)$, $\cos(\phi)$ and $\tan(\phi)$.



(3) A vector \vec{A} is given by $\vec{A}=5\hat{i}+4\hat{j}$. Find the following:

- (a) what is the magnitude of \vec{A} ?
- (b) what is the angle the vector makes with the x-axis?
- (c) what is the angle the vector makes with the y-axis?
- (d) Express this vector using the “hat” notation.
- (e) Express this vector using the “x-y” unit vector notation.

(4) Suppose a vector \vec{B} is given by $\vec{B}=3\hat{i}+2\hat{j}$. Find the following:

- (a) What is $2\vec{B}$?
- (b) What is $\vec{B}+\vec{A}$?
- (c) What is $\vec{B}-\vec{A}$?
- (d) What is $\vec{B}\cdot\vec{A}$? (dot product)
- (e) What is the angle made with respect to the positive x-axis by $\vec{B}+\vec{A}$?

(5) Suppose a vector \vec{C} is given by $\vec{C}=8\hat{i}-9\hat{j}$. A person walks along vector \vec{A} , then vector \vec{B} , followed by vector \vec{C} . At the end of this journey, what is the displacement (vector) and the distance from the origin. You may assume all units are in m here.

(1) Which of the following equations are dimensionally correct? Note: the following quantities have the following dimensions:

$$F: \left[\frac{ML}{t^2} \right]; m: [M]; a: \left[\frac{L}{t^2} \right]; t: [t]; x [L]; E: \left[\frac{ML^2}{t^2} \right]; v: \left[\frac{L}{t} \right]; s: [L]$$

- (a) $F=ma$
- (b) $X=(1/2)at^3$
- (c) $E=(1/2)mv$
- (d) $E=\max$
- (e) $v = \sqrt{\frac{Fs}{m}}$

Solution:

(a) yes: $F = \left[\frac{ML}{t^2} \right]$ as dimensions is correct. $m = [M]$ and $a = \left[\frac{L}{t^2} \right]$, So you can see

that $\left[\frac{ML}{t^2} \right] = [M] \left[\frac{L}{t^2} \right]$ which is in fact Newton's law $\vec{F} = m\vec{a}$.

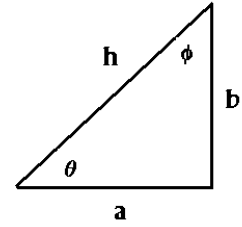
(b) no: $x = [L]$, $a = \left[\frac{L}{t^2} \right]$ and $t = [t]$ so $[L] \neq [Lt]$

(c) no: $E = \left[\frac{ML^2}{t^2} \right]$ and $m = [M]$, $v = \left[\frac{L}{t} \right]$ so $\left[\frac{ML^2}{t^2} \right] \neq \left[\frac{ML}{t} \right]$

(d) yes: $E = \left[M \frac{L^2}{t^2} \right]$, $m = [M]$, $a = \left[\frac{L}{t^2} \right]$, $x = [L]$ so $\left[\frac{ML^2}{t^2} \right] = [M] \left[\frac{L}{t^2} \right]$

(e) yes: $v = \left[\frac{L}{t} \right]$, $F = \left[\frac{ML}{t^2} \right]$, $s = [L]$, $m = [M]$ so $\left[\frac{L}{t} \right] = \left[\frac{L^2}{t^2} \right]^{1/2}$

(2) In the right triangle shown, find the following: h in terms of a and b . Then find $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$, $\sin(\phi)$, $\cos(\phi)$ and $\tan(\phi)$.



Solution:

(a) $h = \sqrt{a^2 + b^2}$

(b) $\sin(\theta) = \frac{b}{h}$ $\cos(\theta) = \frac{a}{h}$ $\tan(\theta) = \frac{b}{a}$

(c) $\sin(\phi) = \frac{a}{h}$ $\cos(\phi) = \frac{b}{h}$ $\tan(\phi) = \frac{a}{b}$

(3) A vector \vec{A} is given by $\vec{A}=5\hat{i}+4\hat{j}$. Find the following:

(a) what is the magnitude of \vec{A} ?

(b) what is the angle the vector makes with the x-axis?

(c) what is the angle the vector makes with the y-axis?

(d) Express this vector using the “hat” notation.

(e) Express this vector using the “x-y” unit vector notation.

Solution:

(a) $|\vec{A}|=\sqrt{(\vec{A}\cdot\vec{A})}=[25+16]^{\frac{1}{2}}=\sqrt{41}=6.403.$

(b)

$$\cos(\theta_{A,x})=\frac{\vec{A}\cdot\hat{x}}{|\vec{A}|}=\frac{(5\hat{i}+4\hat{j})\cdot(\hat{x}+0\hat{y})}{\sqrt{5^2+4^2}}=\frac{5}{6.403}=0.7809\Rightarrow\theta_{A,x}=\cos^{-1}(0.7809)=38.66^\circ$$

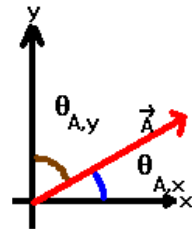
(c)

$$\cos(\theta_{A,y})=\frac{\vec{A}\cdot\hat{y}}{|\vec{A}|}=\frac{(5\hat{i}+4\hat{j})\cdot(0\hat{x}+\hat{y})}{\sqrt{5^2+4^2}}=\frac{4}{6.403}=0.6247\Rightarrow\theta_{A,y}=\cos^{-1}(0.6247)=51.34^\circ$$

My unusual notation for these angles is for clarity: these are angles with respect to the particular axis of interest. Also, note that these 2 angles add up to be 90 degrees.

(d) $\vec{A}=5\hat{i}+4\hat{j}$

(e) $\vec{A}=5\hat{x}+4\hat{y}$



(4) Suppose a vector \vec{B} is given by $\vec{B}=3\hat{i}+2\hat{j}$. Find the following:

(a) What is $2\vec{B}$?

(b) What is $\vec{B}+\vec{A}$?

(c) What is $\vec{B}-\vec{A}$?

(d) What is $\vec{B}\cdot\vec{A}$? (dot product)

(e) What is the angle made with respect to the positive x-axis by $\vec{B}+\vec{A}$?

Solution:

$$(a) \quad 2\vec{B}=(2 \times 3)\hat{i}+(2 \times 2)\hat{j}=6\hat{i}+4\hat{j}$$

$$(b) \quad \vec{B}+\vec{A}=[3\hat{i}+2\hat{j}]+[5\hat{i}+4\hat{j}]=[3+5]\hat{i}+[2+4]\hat{j}=8\hat{i}+6\hat{j}$$

$$(c) \quad \vec{B}-\vec{A}=[3\hat{i}+2\hat{j}]-[5\hat{i}+4\hat{j}]=[3-5]\hat{i}+[2-4]\hat{j}=-2\hat{i}-2\hat{j}$$

$$(d) \quad \vec{B}\cdot\vec{A}=[3\hat{i}+2\hat{j}]\cdot[5\hat{i}+4\hat{j}]=[3 \times 5]+[2 \times 4]=15+8=23$$

$$(e) \quad \cos(\theta_{\vec{B}+\vec{A},x})=\frac{\vec{B}+\vec{A}}{|\vec{B}+\vec{A}|}\cdot\hat{i}=\frac{[8\hat{i}+6\hat{j}]}{\sqrt{8^2+6^2}}\cdot\hat{i}=\frac{8}{\sqrt{100}}=\frac{8}{10}=0.8\Rightarrow\theta=36.87^\circ$$

Note that the power of the method in (e) gives you a manner to obtain the angle with respect to any unit vector. You can define a unit vector in an arbitrary direction easily as:

$$\hat{B}=\frac{\vec{B}}{|\vec{B}|}$$

For example, for this particular vector, we have:

$$\hat{B}=\frac{3\hat{i}+2\hat{j}}{\sqrt{3^2+2^2}}=\frac{3}{\sqrt{13}}\hat{i}+\frac{2}{\sqrt{13}}\hat{j}=0.832\hat{i}+0.555\hat{j}$$

To find the angle between the vectors A and B, you would then take, for example:

$$\begin{aligned}\cos(\theta_{\vec{A},\vec{B}})&=\frac{\vec{A}}{|\vec{A}|}\cdot\frac{\vec{B}}{|\vec{B}|}=\hat{A}\cdot\hat{B}=\frac{[5\hat{i}+4\hat{j}]}{\sqrt{5^2+4^2}}\cdot[0.832\hat{i}+0.555\hat{j}]=\frac{[5\hat{i}+4\hat{j}]}{\sqrt{41}}\cdot[0.832\hat{i}+0.555\hat{j}] \\ &=\frac{4.16+2.22}{\sqrt{41}}=0.996\Rightarrow\theta=4.87^\circ\end{aligned}$$

You could, of course, use the law of cosines which states:

$$\cos(\theta_{\vec{A},\vec{B}})=\frac{-|\vec{B}-\vec{A}|-|\vec{A}|^2-|\vec{B}|^2}{2|\vec{A}||\vec{B}|}=\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}=\hat{A}\cdot\hat{B}=\frac{\vec{A}\cdot\vec{B}}{|\vec{A}|}$$

where I have used the fact that the vector pointing from \vec{A} to \vec{B} is given by $\vec{B}-\vec{A}$.

(5) Suppose a vector \vec{C} is given by $\vec{C}=8\hat{i}-9\hat{j}$. A person walks along vector \vec{A} , then vector \vec{B} , followed by vector \vec{C} . At the end of this journey, what is the displacement (vector) and the distance from the origin. You may assume all units are in m here. $\vec{A}=5\hat{i}+4\hat{j}$ $\vec{B}=3\hat{i}+2\hat{j}$

Solution:

$$\vec{D}=\vec{A}+\vec{B}+\vec{C}=[5\hat{i}+4\hat{j}]+[3\hat{i}+2\hat{j}]+[8\hat{i}-9\hat{j}]=(5+3+8)\hat{i}+(4+2-9)\hat{j}=16\hat{i}+(-3)\hat{j}$$

$$|\vec{D}|=\sqrt{\vec{D}\cdot\vec{D}}=\sqrt{16^2+3^2}=\sqrt{256+9}=\sqrt{265}=15.279$$

You can easily verify that $\vec{A}+\vec{B}+\vec{C}-\vec{D}=\vec{0}$.

The vector symbol over 0 is necessary because the result of adding two vectors must produce a vector.