Kinematics in 1 and 2 dimensions Pandemic version Lab 02

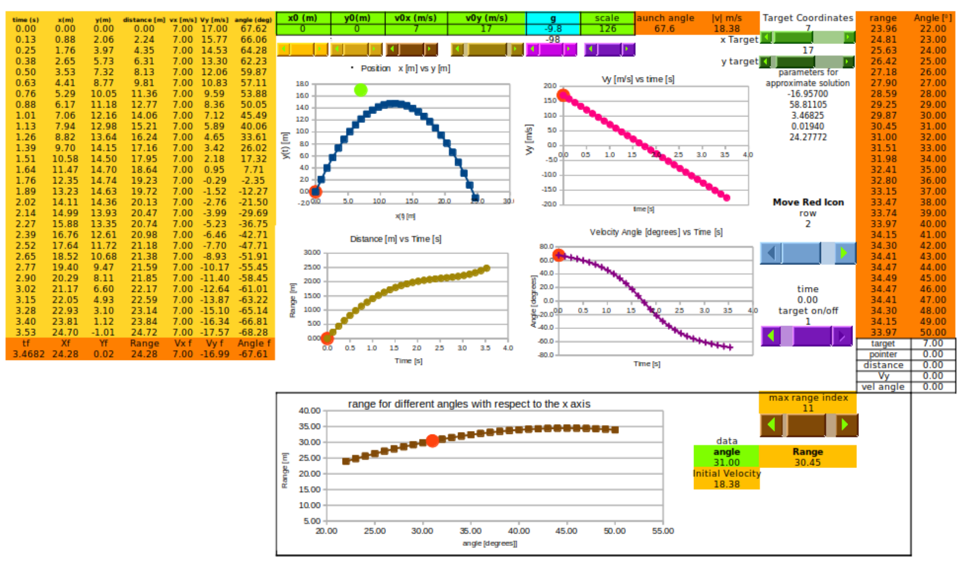
In this lab we are going to simulate the equations of motion under the condition of freefall. The equations of motion are given by:

$$\text{for the x direction:} \quad x = x_0 + v_{0,x} t : v_x = v_{0,x} : a_x = 0$$

$$\text{for the y direction:} \quad y = y_0 + v_{0,y} t - \frac{1}{2} g t^2 : v_y = v_{0,y} - g t : a_y = -g$$

In the spreadsheet simulation of freefall, (shown below) you have the following quantities plotted:

Position (y vs x)
Velocity (V_y vs time)
distance ($\sqrt{x^2+y^2}$ vs time)
Velocity angle (angle w/r to x vs time).
Range (at $\text{speed} = \sqrt{v_{0,x}^2 + v_{0,y}^2}$ vs angle).



The goal that you have for today's lab is to understand how variations in initial parameters influence the outcomes of the various quantities. You are also able to solve (approximately) 4th order polynomials.

I have written these lab instructions to closely model exactly what would happen in the lab under normal circumstances. The physical lab for this is done in the parking lot on the east side of the library and I will refer to this in this simulation. In your calculations below, be sure to clearly include proper SI units in your work.

Part 1: find the velocity of water leaving the squirt gun.

You will first need to find the velocity that the water leaves the gun with. In the first simulation (named: pandemic1d-modified.ods), simulate using the following initial conditions: x_0=.2 m, a_y=-9.8 m/s^2, t_0=0 s. You need to walk through the simulation to find the maximum height that is observed. You will need to change the scale also. At the maximum height, the velocity is at a minimum and in fact, for one dimensional freefall, the velocity is zero. This is, in fact the simulation of squirting the gun straight up , measuring the maximum height and from there finding the speed that the water leaves the gun with.

Write down this maximum height. You can use the brown control to move the pointer through the collected data. You might imagine that the data is a photograph of the water moving upward. Include a screen capture showing this maximum height in your report.

Now you need to determine (this is a calculation that you will do by hand on paper and include in your lab report) what the initial velocity was that the water left the gun with.

This is determined by: $v^2=v_0^2-2g\Delta x$ where g = 9.8 m/s^2. Be sure that you use Δx and not just x. You will need this velocity for the second part of the experiment.

Part 2: determination of the range

In the physical lab, after measuring the initial velocity that the water leaves the gun with, a measurement of the height of the balcony above the parking lot is required. This height is measured to be 4.7 m in previous years.

The squirt gun is taken to the balcony and shot parallel to the ground and the distance between the gun and where the water hits the parking lot is then measured. You are now to use your velocity that you measured in part 1 to determine this distance following the steps below (again this is a hand calculation that you include with your report.)

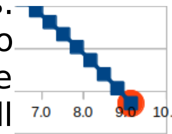
The time of fall from the balcony is given by the first equation for y:

$$y=y_0+v_{0,y}t-\frac{1}{2}gt^2 \quad .$$

Since the gun is shot parallel to the parking lot, $v_{0,y}$=0. The water falls through a distance of ($y-y_0$)=(0 – 4.7)=-4.7 m. So now find the time of fall. You will choose the positive solution here.

Next, in this time of fall, the water moves a distance of Δx from the balcony. This is given by the first equation of motion for x: $x = x_0 + v_{0,x} t$ where the time here is the time of fall you determined above. You may assume that x_0 here is zero. So now determine how far from the balcony the water will be when it hits the ground.

Now it is time to do the second simulation to check your results. So open the second spreadsheet (Pandemic-lab02FreeFallSimulationModified.ods) and enter your values to verify that the expected result coincides with the predicted values. You will need to change the scale in order to get a bracket of the last two position values. It looks like the image to the right when you are in the best position to get the approximate solution which is contained in cell B32. A note on the two simulations: I have modified both of these a bit to fit better with the lab. The full spreadsheet simulations will be linked on the class pages. As a note: the approximate solution is obtained by a linear fit to the last two points and then solving this fit for the point at which zero is obtained.



You should now (by hand) calculate the percent error given by:

$$\%\,\text{error} = 100\,\text{x}\,\frac{[\text{measured} - \text{expected}]}{\text{expected}} \quad .$$

Note that this error can be positive or negative depending on if the measured result is larger or smaller than the expected result.

Now it is possible to do the range calculation with just the h_{stick} and h_{gun} in the following way: The initial velocity is given by $V_{initial} = \sqrt{2gh_{stick}}$ which is the velocity that the water has when leaving the gun and h_{stick} is the maximum height of the water column. The time of fall is given by: $y_f = y_0 + v_{0,y} t - \frac{1}{2} g t^2 \Rightarrow t = \pm\sqrt{\frac{2h_{gun}}{g}}$ where h_{gun} is the height the water gun is above the parking lot. If we use these two results in the x eqation of motion we have $x = v_{0,x} t = \sqrt{2gh_{stick}}\sqrt{\frac{2h_{gun}}{g}} = 2\sqrt{h_{stick} h_{gun}}$. Work through this and include this (hand calculation) in your report.

Part 3: the maximum range is at 45 degrees

The spreadsheet simulation also plots for a constant initial velocity, the range that is expected for different angles of projection. In order to use this, you will have to set your initial position equal to zero for both x and y in the simulation. Strictly said, the range equation will only apply to an object that undergoes freefall and returns to it's initial altitude. This is why you need to set the initial x and y positions to zero.
This is the plot at the bottom of your simulation. You can leave the initial velocity as it is but the same results will happen if you choose to use different values.

If you remember from class, the time of fall can be obtained in the following way: the magnitude of the initial velocity is given by:

$$|\vec{v}_0| = \sqrt{v_{0,x}^2 + v_{0,y}^2} \equiv v_0$$

Trignometry then gives us (where Θ is the angle with respect to the x-axis):

$$v_{0,x} = v_0 \cos(\theta) : v_{0,y} = v_0 \sin(\theta)$$

The time of fall is given by:

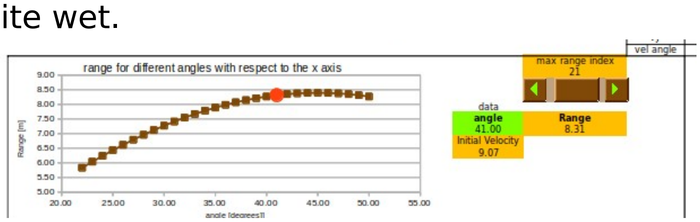
$$v_y = v_{0,y} - gt : v_y = -v_{0,y} : \Rightarrow -v_{0,y} = v_{0,y} - gt \Rightarrow t = \frac{2v_{0,y}}{g} = \frac{2v_0 \sin(\theta)}{g}$$

The range is then given by:

$$x = v_{0,x} t = [v_0 \cos(\theta)] \left[\frac{2v_0 \sin(\theta)}{g} \right] = \frac{v_0^2}{g} (2\sin(\theta)\cos(\theta)) = \frac{v_0^2 \sin(2\theta)}{g}$$

For the maximum range to occur, we need to maximize $\sin(2\theta)$. This will be when $\theta = 45^0$.

In the simulation, you are now to scroll through different angles of projection for a constant initial velocity to observe the angle at which the maximum range occurs. In the physical lab, this is done by squirting the gun, raising it up while turning and observing the maximum range that results. Also it is not uncommon for students to come out of this lab quite wet.



As part of your lab report today, make and include sketches of what would happen in the physical lab. As part of your lab report, I also want you to write sentences about what the other 3 graphs are showing for a free fall situation and discuss intersections and inflections that are observed in these plots. Do note that the last data point is from the approximate solution to the equations of motion so it may appear misplaced, but it is not. Also in the distance vs time plot, the distance is calculated from : $distance=\sqrt{x^2+y^2}$ and is not quite the same as range (normally).

Physical lab description

Everything below this point is to help you make a connection between the physical lab and the simulation.

I am including this description of what our simulation is simulating if we were in the lab doing the experiment to help you describe the connection between reality and the simulation in your methods portion of your lab report.

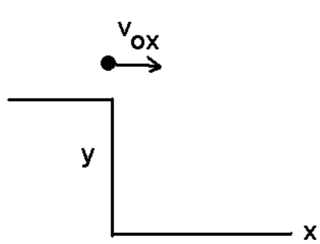
Fluid motion is most correctly discussed using Bernoulli's equation and using this as an example of conservation of energy. Today, however, we are taking a slightly different approach. In particular, I will not be using Bernoulli's equation since it has been my experience that students miss the point that this lab is really about which is 1 and 2 dimensional motion.

If you treat a stream of water as if it was composed of small non-interacting particles, each with a mass m, then you can imagine that a stream of water is composed of just such particles … you might think that we are checking the validity of using the equations of motion when treating water as a stream of particles. Today, the stream of particles will be produced by a squirt gun. Let's now talk about aspects of this experiment which you are more familiar with. The small pellets of water each have an initial velocity $\vec{v}_0$. So long as we direct this either along the x or y direction, we can ignore the vector quality for this discussion. Let's answer the question of what is v_0. **Imagine you direct the gun vertically and the water is observed to achieve a maximum height h relative to the point where the water left the gun**. The third equation of motion is:

$$v^2 = v_0^2 - 2g(\Delta y) \Rightarrow v_0 = \pm\sqrt{2gh}$$

The correct sign choice here is the positive sign. You will have noted that at the point of maximum altitude, the water has zero velocity. Thus you are able to determine the initial velocity of the water leaving the gun by measuring the maximum height of the vertical water column when the gun is directed upward.

Now that you understand how to obtain the initial velocity, this can be used to determine the range of the water when shot horizontally. You can imagine, if you like, that the water gun is shooting out little pellets of water, one after another also in the second experiment.



Suppose a rock is an initial height y_0 above the ground and is thrown with an initial velocity $v_{o,x}$ in the x-direction. The question is how far from x_0 will the rock be when it strikes the ground (at y=0)? This is a fundamental type of problem you should be able to work as a result of the physics class but we'll go through the solution here again.

For motion in the x-direction, we have

$$x = x_0 + v_{0,x} t$$

where t is time **and we assume no acceleration in the x-direction**. For motion in the y-direction, we have:

$$y = y_0 + v_{0,y} t - \frac{1}{2} g t^2$$

Now, suppose that your do the following sequence of experiments: (1) Throw a ball straight up with an initial velocity $V_{0,y}$ (I am using capital letters for v here to reduce confusion. How high does the ball go?

$$V_y^2 = V_{0,y}^2 - 2g(\Delta y) \Rightarrow \Delta y = \frac{V_{0,y}^2}{2g}$$

.

Now what this means is that if we measured how high the ball went, we can obtain the initial velocity as:

$$V_{0,y} = \pm\sqrt{2g\Delta y}$$

Let's call this $\Delta y = h_{stick}$ since it is measured with a long stick.

Now for the next part of the experiment: you now know how fast water leaves the gun after this measurement and calculation. It is given by:

$$V_{initial} = \sqrt{2 g h_{stick}}$$

Suppose that you now squirt your gun along the x-direction off of the top of the balcony of the library. There is now no initial velocity in the y direction: it is all in the x direction. Further, suppose that the distance from the ground to the gun is given by h_{gun} . How long does it take for the water to hit the ground from this height (which is essentially the same question as how long does it take a ball to fall through this distance when released from rest). The answer to this is given by:

$$y_f = y_0 + v_{0,y} t - \frac{1}{2} g t^2 \Rightarrow t = \pm\sqrt{\frac{2 h_{gun}}{g}} \quad .$$

Now you can answer the question of how far in the x-direction will the stream of water shoot until it hits the ground. This is given by:

$$\Delta x = v_{0,x} t = \sqrt{2 g h_{stick}} \sqrt{\frac{2 h_{gun}}{g}}$$

We can combine these into one single result:

$$\Delta x = 2\sqrt{h_{stick} h_{gun}} \quad .$$

This is a simple enough result that you don't need a spreadsheet really to calculate it.

Using your measured h_{stick} and h_{gun}, calculate Δx (this is the theoretical value) before you actually fire your water gun from the library balcony.

$\Delta x_{expected}$ = _______________ m

Show me your theoretical calculation before you actually spray water from the balcony. Then you'll fire your gun and obtain $\Delta x_{measured}$.

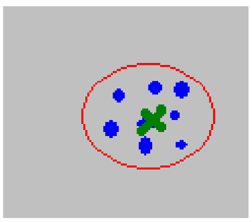
Equipment

Big Stick: In order to measure h_{stick}, I have located a long board. You might consider marking the board with colored chalk to record your measurement here.

Tape measure: In order to measure , use the tape measure I have provided. You'll also use the tape measure ultimately to measure h_{gun}.

Water gun: One Black Widow high pressure water gun is needed for obvious reasons.

Chalk: You'll want to mark the water spots that came from your gun. Chalk is probably the thing that you need to do this with. Do it quickly before the water dries. You will want to imagine a circle around the water droplets and take the center of the circle as if it were the point where the water struck the ground.



Procedure

The best location for the height measurements of the water column is in a small corner located at the edge of the library. Fill your water gun with water and then pump it up until the maximum pressure is reached. You will see a bit of water start to drip out of the gun at this point. Align the long stick along the wall and make a quick release **(hint: don't stand up)** on the water gun, allowing the water to strike the wall. Do not move the water gun until the gun's position has been recorded. With the long stick, measure the distance Δh. Give short bursts trying to keep the pressure the same in all experiments. You will want to count how many pumps were required for experiment 1 and keep the water level the same (most guns have an indicator on them, and additional water is available).

Next, one lab partner should go to the balcony of the library with the water gun. Measure the height of the gun above the roadway. Align the nozzle of the gun with the edge of the library so that the water shoots out without any initial y-velocity. When the gun is fully charged again, release a short burst along the road. Your lab partner should mark with colored chalk the point where the water hits the road. Measure this distance from the position directly under the gun to the chalk mark with the measuring tape. You will note that the water lands in a dispersion pattern. You probably want to measure the center and about 2/3 of the way from the center to the droplets of water represent the ± experimental error.

You should do the entire experiment three times since a small amount of wind can disturb your results. **Hint: wait until there is no wind to shoot the gun.**

After you complete your experiment, be sure to empty the water guns and observe the trajectory which the water takes once it leaves the gun. The kinematic equations of motion clearly predict this type of motion as you know from class. **In particular, I**

recommend testing for the maximum range corresponding to an angle of 45^0.

Analysis

	Experiment 1	Experiment 2	Experiment 3
h_{stick} [m] (1 time)			
h_{gun} [m] (1 time)			
Δx (m)			
Δx (m) expected			
% error			

You may calculate the % error by:

$$\%\text{Error} = \frac{\text{measured} - \text{expected}}{\text{expected}} \times 100 \quad .$$

This definition is such that if the measured value is less than the expected value, a negative % error results; otherwise a positive % error results. You may notice that your error is on the order of 15% or so. You should discuss some reasons that this might be the case in your analysis. **Your analysis should also repeat the derivation that I presented above in order to determine the distance the water travels.**

Conclusion

Your conclusion should discuss the equations of motion which were applicable here. You should also make a sketch of the trajectory which the water takes leaving the gun. You should come away from this lab with a better understanding of application of the equations of motion in two dimensions.