Gallilean (classical) transformations

$$\begin{vmatrix} x'=x-vt \\ y'=y \\ z'=z \\ t'=t \end{vmatrix} \quad \text{Inverse:} \quad \begin{vmatrix} x=x'+vt' \\ y=y' \\ z=z' \\ t=t' \end{vmatrix}$$

Time/length dilation

$$t=\gamma t_0 : L=\frac{1}{\gamma}L_0$$

$$\beta\equiv\frac{v}{c} : \gamma\equiv\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{1}{\sqrt{1-\beta^2}}$$

Lorentz Transformations

$$\begin{vmatrix} x'=\gamma(x-vt) \\ y'=y \\ z'=z \\ t'=\gamma\left(t-\frac{vx}{c^2}\right) \end{vmatrix} \quad \text{Inverse:} \quad \begin{vmatrix} x=\gamma(x'+vt') \\ y=y' \\ z=z' \\ t=\gamma\left(t'+\frac{vx'}{c^2}\right) \end{vmatrix}$$

Contravarient vector

$$X^{\mu}=(x^0,x^1,x^2,x^3)=(ct,x,y,z)$$

Covarient vector

$$X_{\mu}=(x_0,x_1,x_{2,}x_3)=(-ct,x,y,z)$$

“boost” transformation

$$\begin{pmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{pmatrix}=\begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

Einstein summation convention

$$a_{\mu}b^{\mu}\equiv\sum_{\mu=0}^{\mu=4}a_{\mu}b^{\mu}=-a^0b^0+a^1b^1+a^2b^2+a^3b^3$$

(Invariant)

The Invariant Interval

$$\text{event } A : X_A = (x_A^0, x_A^1, x_A^2, x_A^3) :: \text{event } B\, X_B = (x_B^0, x_B^1, x_B^2, x_B^3)$$

Define: $\Delta X^\mu \equiv X_A^\mu - X_B^\mu$

The interval between the two events is given by:

$$I \equiv -(\Delta x^0)^2 + (\Delta x^1)^2 + (\Delta x^2)^2 + (\Delta x^3)^2$$

I<0 : time like: two events can occur at the same place

I>0: space-like: two events can occur at the same time

I=0: two events can only be connected by light signal

Relativistic Velocity addition formula

start with:

$$\left| \begin{matrix} x' = \gamma(x - vt) \\ y' = y \\ z' = z \\ t' = \gamma\left(t - \frac{vx}{c^2}\right) \end{matrix} \right|$$

Assume inertial frames

$$\begin{matrix} dx' = \gamma(dx - v\, dt) \\ dy' = dy \\ dz' = dz \\ dt' = \gamma\left(dt - \frac{v\, dx}{c^2}\right) \end{matrix}$$

$$u'_x = \frac{dx'}{dt'} = \frac{dx - v\, dt}{dt - \frac{v}{c^2}\, dx} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u'_y = \frac{dy'}{dt'} = \frac{dy}{dt - \frac{v}{c^2} dx} = \frac{u_y}{1 - \frac{u_x v}{c^2}}$$

$$u'_y = \frac{dy'}{dt'} = \frac{dy}{dt - \frac{v}{c^2} dx} = \frac{u_y}{1 - \frac{u_x v}{c^2}}$$